Optimal Survey design using the point spread function measure of resolution

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Summary

An objective in survey design is to determine optimal survey parameters, such as the position of sources/receivers and possibly frequencies in EM experiments, that would provide 'better' model resolution in a region of interest. We pose survey design as an inverse problem by maximizing a resolution measure, the point spread function. The point spread function quantifies how an impulse in the true model is observed in the inversion result and, hence, the goal is to adjust the survey parameters so that the point spread function is as delta-like as possible. This problem is solved as a nonlinear optimization problem with constraints on the parameters. Due to the highly nonlinear nature of the problem we examine two approaches for its solution. The first is a local, (Newton) strategy that use a primal interior point method to incorporate bounds on the parameters; the second is global method, simulated annealing. This paper primarily concentrates on the problem formulation and we illustrate our methodology through application to a ray-based tomography example.

Introduction

The idea of 'better' survey design is not new in geophysics. The traditional approach of carrying out a reconnaissance survey (typically an airborne survey) followed by a ground survey has been routinely applied in mineral and oil exploration problems. The general idea is to focus on a region that shows anomalous values in the large scale survey and follow it up with a detailed ground survey. The interpretation of ground survey data is then validated by borehole measurements. Essentially, this approach is data driven, where the survey area is spatially scaled down to enhance the anomalous response.

A different way to look at the problem is: "Given a region of interest where the target is most likely to occur, can we design a survey to provide enhanced resolution in the region of interest?" The ability to achieve this goal would allow us to obtain better images of the target. Depending upon what type of data are acquired, the survey design objective can be different. For example, it can be the determination of appropriate spatial locations of sources and receivers in a seismic or electrical tomography experiment, or the determination of appropriate frequency and temporal sampling in an electromagnetic or seismic experiment. The problem has received considerable attention in recent years with the goals of reducing the cost of acquisition and processing and enhancing model resolution. The basic approach has been to formulate survey design as an optimization problem (Maurer and Boerner, 1998). From previous work, it is well recognized that this is an extremely nonlinear problem and, therefore, the commonly applied methods are global optimization techniques such as genetic algorithms and Monte Carlo searches to determine the optimal survey parameters (Maurer and Boerner, 1998). The nonlinearity depends on the choice of the cost function in the optimization problem. Curtis (1999b; 1999a) maximizes positivity measures of eigenvalues of the operator G^TG using a generic algorithm to design optimal surveys geometries in a cross-well seismic experiment. We take a different approach in choosing the objective function and maximize a resolution measure called the point spread function.

Methodology

Our approach in setting up the optimization problem is as follows: First we solve the inverse problem with a given geometry and obtain a model that we denote as the primal inverse problem. For a linear problem this is achieved by minimizing a quadratic objective function

$$\phi(m) = \beta \|W_m (m - m_0)\|^2 + \|W_d \left(d^{obs} - Gm\right)\|^2. \quad (1)$$

The estimated model $\hat{m}$ is given by

$$\hat{m} = A^{-1}\left(G^T W_d^T W_d d^{obs} + \beta W_m^T W_m m_0\right) \quad (2)$$

where $A = \left(G^T W_d^T W_d G + \beta W_m^T W_m\right)$, W_m, W_d are model and data weighting matrices, β is the regularization parameter, $G \in \mathcal{R}^{N\times M}$, $m \in \mathcal{R}^M$, and $d^{obs} \in \mathcal{R}^N$. The resolution matrix that provides a mapping between the estimated model $\hat{m}$ and the true model m, can be obtained by replacing $d^{obs} = Gm + \epsilon$ in eq(2). This is given by

$$R = \left(G^T W_d^T W_d G + \beta W_m^T W_m\right)^{-1}\left(G^T W_d^T W_d G\right). \quad (3)$$

We are particularly interested in the columns of the resolution matrix R called the point spread function, PSF. A PSF describes how a delta-like perturbation in the model manifests itself in the inverted result. If a particular PSF is wide and/or has side lobes, then the corresponding model is poorly resolved. For large scale problems the PSF can be obtained without explicitly forming the resolution matrix R. Instead we can minimize

$$\begin{pmatrix} W_d G \\ \sqrt{\beta} W_m \end{pmatrix} (p_k) = \begin{pmatrix} W_d G \Delta_k \\ 0 \end{pmatrix} \quad (4)$$

in a least-squares sense using a conjugate gradient type iterative solver like CGLS. In eq(4) Δ_k is a vector that is zero everywhere except at the k^{th} cell; it is an impulse function. The PSF p_k in eq(4) has the same dimensionality as the model i.e., $p_k \in \mathcal{R}^M$. Although eq(4) does not indicate the explicit dependence on noise and model discretization, we recognize that the choice of regularization parameter β is implicitly dependent on data noise and model discretization. Similarly, the explicit dependence on survey parameters is not exhibited by eq(4), however the kernel matrix G is a function of survey parameters. For example, G can be a function of receiver position r, source positions r_s, frequencies ω for a frequency domain survey or times t for a time domain survey. Equation (4) clearly establishes a nonlinear dependence of a PSF on the survey parameters. The character of a PSF will be location dependent, thus they provide us with the ability to study the resolving capability of a survey for a localized region of the model. This is the central role of a PSF in 'optimal' survey design. The problem can be approached in two ways: (a) We can compare the PSF's from different surveys to determine which survey best resolves our region of interest or (b) we can formulate an optimization problem to determine survey parameters that will maximize the PSF in the region of interest. In this paper we concentrate on the optimization problem approach.

We formulate the optimization problem by minimizing the closeness of the PSF to the impulse function Δ_k subject to the constraint that the survey parameters are within some bounds. The bounds can be imposed based on the physical limitations of the acquisition system and logistic procedures. Denoting the survey parameters by ξ, the inverse problem can be mathematically stated as

$$\min \ ||W_k\,(p_k(\xi) - \Delta_k)||^2 \quad \text{s.t.} \quad \xi_{min} \le \xi \le \xi_{max} \qquad (5)$$

where the diagonal weighting matrix W_k is given by

$$(W_k)_{jj} = 1 + |r_j - r_k|^\gamma \ (\gamma > 0) \text{ for } r_j \ne r_k \qquad (6a)$$
$$= \alpha \text{ for } \ r_j = r_k. \qquad (6b)$$

W_k is a distance weighting matrix that penalizes the side lobes of the PSF away from the k^{th} cell for $r_j \ne r_k$. The value of γ controls the rate at which energy in the PSF away from the cell of interest is penalized. At $r_j = r_k$, α is chosen to be a large number to enhance the amplitude of the PSF and also prevent the overall PSF from approaching zero.

Minimization using a local method: Interior Point Method

The bounded optimization problem in eq(5) can be solved using an interior-point optimization method (Li and Oldenburg, 2000) by minimizing

$$\psi(r) = \frac{1}{2}\,||W_k\,(p_k(\xi) - \Delta_k)||^2 \qquad (7a)$$
$$-\lambda \sum_{k=1}^{N} \log\left(1 - \frac{\xi_k}{\xi^{max}}\right) - \lambda \sum_{k=1}^{N} \log\left(\frac{\xi_k - \xi^{min}}{\xi^{max}}\right) \qquad (7b)$$

where the logarithmic terms in eq(7) increase the value of the objective function when the parameters are close to their bounds, thereby creating a barrier. The log-barrier parameter λ controls the contribution of the log terms in the bounded minimization problem. We linearize $\psi(\xi+\delta\xi)$ about a current model ξ with a Taylor series expansion and minimize the resulting equation with respect to the perturbation i.e., $\nabla_{\delta\xi}\psi(\xi + \delta\xi) = 0$. Denoting $X = \text{diag}(1/(\xi - \xi^{min}))$ and $Y = \text{diag}(1/(\xi^{max} - \xi))$ the resulting system of equations after minimization is given by

$$\left(J^T W_k^T W_k J + \lambda X^2 + \lambda Y^2\right) \delta\xi = J^T W_k^T W_k \delta d \qquad (8a)$$
$$+\lambda X - \lambda Y \qquad (8b)$$

where $\delta d = \Delta_k - p_k(\xi)$ is the residual error and $J \in \mathcal{R}^{M\times N}$ is the sensitivity matrix of the PSF with respect to the survey parameters ξ. The solution to the system of eq(8) can be obtained by minimizing

$$\begin{pmatrix} W_k J \\ \sqrt{\lambda}X \\ \sqrt{\lambda}Y \end{pmatrix} [\delta\xi] = \begin{pmatrix} W_k \delta d \\ \sqrt{\lambda}e \\ -\sqrt{\lambda}e \end{pmatrix} \qquad (9)$$

in a least-squares sense where $e \in \mathcal{R}^N$ is a vector of ones. To solve for perturbation $\delta\xi$ we need to compute the sensitivity. The elements of the sensitivity matrix J are computed using a forward finite-difference method. The perturbed solution to the PSF $p_k(\xi_j + \delta\xi_j)$ is obtained by first computing the perturbed kernel matrix $G(\xi_j + \delta\xi_j)$ and solving

$$\begin{pmatrix} W_d G(\xi_j + \delta\xi_j) \\ \sqrt{\beta} W_m \end{pmatrix} (\, p_k(\xi_j + \delta\xi_j\,)) = \qquad (10a)$$
$$\begin{pmatrix} W_d G(\xi_j + \delta\xi_j)\Delta_k \\ 0 \end{pmatrix} \qquad (10b)$$

using CGLS. Thus for N survey parameters, calculation of J requires N forward solutions of the eq(10). In solving eq(10) we keep the regularization parameter β that was obtained in the primal inverse problem. Thus a regularization constant that is appropriate for the noise level of the data is included in the analysis. We compute the new PSF using eq(10) and solve for the parameter perturbation using eq(9). The optimization problem in eq(8) is solved by decreasing the barrier parameter λ. The parameter updates are carried out within the barrier iterations using a line search procedure. Because this is a linearized procedure the solution will be at the nearest local minimum from the starting model. This may not be the optimum solution that is sought and, hence, we are motivated to apply a global optimization technique.

Minimization using a global method: Simulated Annealing

We recognize that the inverse problem given in eq(5) is very nonlinear and therefore a global optimization procedure might be superior. We implement a simulated annealing (SA) algorithm to solve eq(5). When minimizing a function using SA, any decrease in the objective function is accepted and the process repeats from this new point. If the objective function value increases, the model may be accepted based on the Metropolis criteria. There are different variants of the algorithm depending on how the acceptance criteria is handled. The specific details of the SA algorithm used here can be found in Routh and Roy (1998). At each iteration a pseudo-parameter called temperature is decreased to effectively decrease the length of the parameter updates. Since the algorithm makes very few assumptions regarding the function to be optimized, it is quite robust with respect to non-convex objective functions. An advantage of using SA is that the sensitivity of the PSF with respect to the survey parameters is not required; forward computation of the PSF is sufficient. However, the main drawback of SA is that it requires many function evaluations. This makes the algorithm expensive to implement if the number of parameters is large.

Tomography Example

To illustrate our survey optimization methodology we consider a cross-well tomography example where the rays are traced from source to receiver through a heterogeneous slowness medium. The cross-well geometry has 13 sources on the left and 19 receivers on the right. We fix the receiver positions and optimize the survey with respect to the source locations. The data, generated with the slowness model in Fig. 1(a), are contaminated with Gaussian random noise that has zero mean and a standard deviation of 5 ms. We solve the primal inverse problem using the initial source locations by minimizing a model objective function given by

$$\phi_m = \alpha_s \, ||W_s(m - m_0)||^2 + \alpha_x \, ||W_x(m - m_0)||^2 + \quad (11a)$$

$$\alpha_z \, ||W_z(m - m_0)||^2 \quad (11b)$$

We chose $\alpha_s = 0.001, \alpha_x = 1, \alpha_z = 1$ for this example. W_s is the smallest weighting matrix and W_x and W_z are flattest weighting matrices that penalize the first derivatives in x and z directions respectively. The initial source locations begin from 35 m and are chosen so that there is insufficient ray coverage in the top left anomalous part of the model. The model from the primal inversion in Fig. 1(b) shows that the anomaly to the left is poorly recovered due to the inadequate ray coverage in the area.

We solve the survey optimization problem by maximizing the PSF for a single cell in the top anomalous region (cell 250) marked with a star in Fig. 1(a). We impose uniform bounds on all source positions such that $0 \leq \xi \leq 100$. The algorithm is terminated when there is no further decrease in objective function value. After optimization, the sources are moved up to increase ray coverage in cell 250 as shown in Fig. 2(b). Comparing Fig. 2(a) and (b), we observe that the total raypath length in cell 250 increases from 16.55 m to 98.10 m and the number of rays crossing the cell increases from 6 to 36. Such increases are intuitively appealing. The corresponding PSF shown in Fig. 3(c) is more localized around cell 250 and the

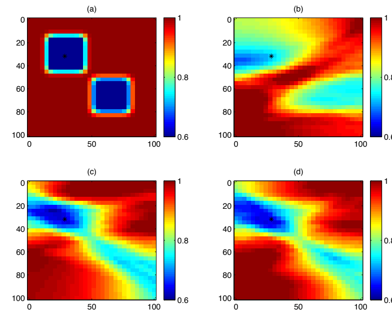

Fig. 1: (a) True and (b) inverted model obtained with initial survey parameters. Inverted models after survey optimization using (c) IPM and (d) SA. The star denotes the position of cell 250.

amplitude has increased by more than 100%. Although in this example we have posed the survey optimization so as to better resolve only a single cell in model space, the idea can be extended to broader region of the model space. Even using the resolution for a single cell, the data acquired with the new survey parameters show improvement in the recovery of the top anomalous body indicated in Fig. 1(b).

Next we apply SA to optimize the location of the source positions. Again, the rays traced with new source locations indicate better ray coverage in the region of interest. Comparing Fig. 2(a) and (c) shows that the total raypath length in cell 250 increases from 16.55 m to 86.00 m and the number of rays crossing the cell increases from 6 to 32. This clearly indicates the improvement achieved by survey optimization and the corresponding PSF shows better localization around the cell. The inverted model with the new survey parameters in Fig. 1(d) shows improvement in the recovery of the top anomalous body. However as the sources move surfaceward to resolve cell 250, the recovery of the bottom anomaly is reduced. This behavior is expected.

To examine the improvement in model estimation process we plot the Backus-Gilbert (BG) averaging function before and after survey optimization, shown in Fig. 4. BG kernels are the rows of the resolution matrix R. The BG kernel function in Fig. 4 for cell 250 indicates significant improvement after survey optimization indicating the reliability of the estimated value.

To illustrate the ability of the survey optimization procedure to resolve different regions in the model, we consider PSF maximization in two different cells. In addition to the cell in top anomalous region we consider maximizing the cell in the bottom anomalous region with the goal of enhancing the resolution of both the anomalies. We maximize the PSF in cells 250 and 590 marked with stars in Fig. 5(a). The new survey parameters result in improved recovery of both the top and bottom anomalous bodies, indicated in Fig. 5(c) and (d). This idea employing multiple PSF's can be extended to obtain enhanced resolution (or focusing) in a region of interest within the model domain.

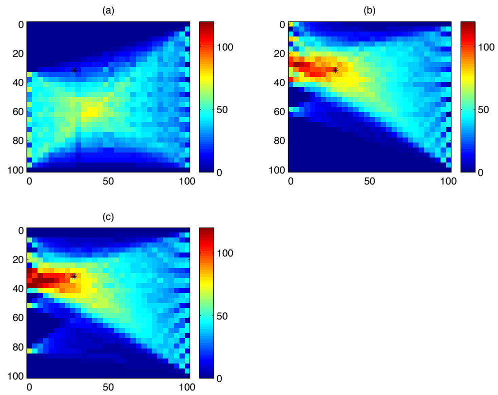

Fig. 2: Raypath length in each cell (a) before and after survey optimization using (b) IPM and (c) SA. The star denotes the position of cell 250.

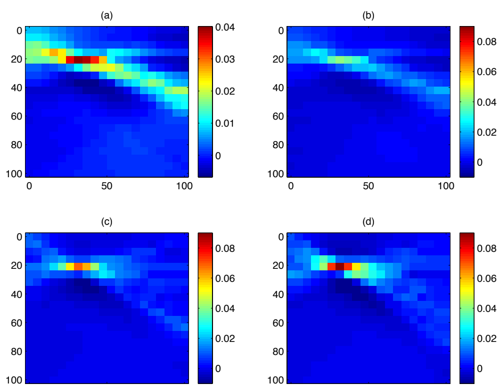

Fig. 3: Point spread functions for cell 250. (a) PSF after the primal inversion stage. Notice the side lobes extending downward. (b) The same PSF in (a) plotted with the scale in (c) and (d). PSF obtained after survey optimization using (c) IPM and (d) SA.

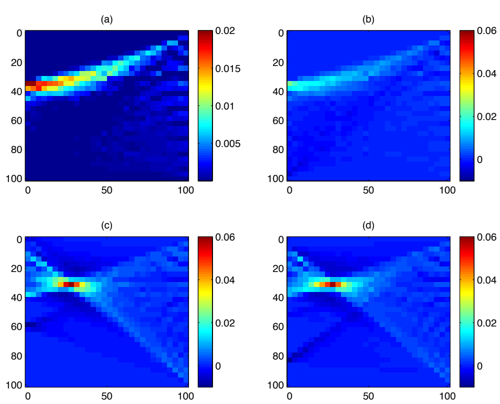

Fig. 4: Backus-Gilbert Averaging functions for cell 250. (a) After the primal inversion stage. (b) The same BG averaging function in (a) plotted with the scale in (c) and (d). BG averaging function obtained after survey optimization using (c) IPM and (d) SA.

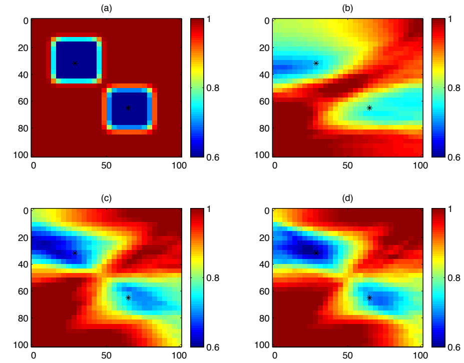

Fig. 5: Multiregion survey optimization. (a) True slowness model. (b) Inverted model obtained with initial survey parameters. Inverted model after survey optimization using (c) IPM and (d) SA. The stars denote the positions of the cells where the PSF's are maximized

Discussions

In this work we have developed the survey optimization problem using a resolution measure derived from appraisal analysis. To determine the optimal survey parameters, we maximize the point spread function in a region of interest. The PSF is a particularly useful measure because it is connected to all of the components that affect resolution: the physics of the problem is buried in the kernel matrix, a priori information is in the model weighting matrix and data noise is effectively handled by regularization and model discretization. Synthetic results indicate that simulated annealing optimization provides better resolution in the region of interest. This is not surprising given the nonlinear nature of the problem. Although this simple example illustrates the methodology in general, we will present examples applying it to electromagnetic problems.

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