# Scale Attributes from Continuous Wavelet Transform

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**Summary**

Average instantaneous attributes of time-frequency decompositions are useful in revealing the time varying spectral properties of seismic data. In the continuous wavelet transform (CWT), a time signal is decomposed into a time-scale spectrum or a scalogram; unlike a time-frequency spectrum or a spectrogram from the short time Fourier transform (STFT). Although there are various approaches of converting a time-scale spectrum into a time-frequency spectrum we introduce new mathematical formulas to calculate spectral attributes from the scalogram. In this process, we bypass the conversion of a scalogram into a time-frequency spectrum and provide average spectral attributes based on scale. The attributes are: center frequency, dominant frequency, and spectral bandwidth. Since these attributes are based on the CWT, computation of these attributes avoids subjective choice of a window length.

**Introduction**

Spectral decomposition of non-stationary signals, like seismic signals, is conventionally achieved by the STFT. Therefore, average instantaneous spectral attributes, such as center frequency, dominant frequency etc., from a spectrogram inherits the properties of the STFT. A relatively new approach of spectral decomposition based on the CWT avoids the subjective choice of a window length and produces a time-scale spectrum also called scalogram. Sinha et al. (2003) converted a scalogram into a time-frequency spectrum called the time-frequency from CWT (TFCWT). One can use the TFCWT to compute instantaneous spectral attributes using the formulas outlined by Barnes (1993). However, we define new formulas to compute the instantaneous spectral attributes directly from a scalogram. In this definition we utilize the fact that the dilating support of the given wavelet, i.e. scale, is inversely proportional to the center frequency of the wavelet (Abry et al., 1993). We use Morlet wavelet for which this mapping is a good approximation.

**Theory**

Instantaneous spectral attributes of center frequency, dominant frequency, and spectral bandwidth are defined as various moments of a time-frequency distribution using familiar definitions from probability theory (Barnes, 1993). Mean of an instantaneous power spectrum is called instantaneous center frequency and is defined as the first moment of the time-frequency power spectrum along the frequency axis. Mathematically, it is given by,

$$f_C(t) = \frac{\int_0^\infty f \left| W(f,t) \right|^2 df}{\int_0^\infty \left| W(f,t) \right|^2 df}, \qquad \text{--- (1)}$$

where $f_C(t)$ is instantaneous center frequency. $W(f,t)$ is the time-frequency map that can be obtained from STFT, TFCWT or any other spectral decomposition.

Square root of the second moment of the time-frequency power spectrum along the frequency axis is called instantaneous dominant frequency.

$$f_D^2(\tau) = \frac{\int_0^\infty f^2 \left| W(f,\tau) \right|^2 df}{\int_0^\infty \left| W(f,\tau) \right|^2 df}, \qquad \text{--- (2)}$$

where $f_D(t)$ is instantaneous dominant frequency.

Standard deviation about the instantaneous center frequency is called instantaneous spectral bandwidth.

$$f_{BW}^2(\tau) = \frac{\int_0^\infty (f - f_C(\tau))^2 \left| W(f,\tau) \right|^2 df}{\int_0^\infty \left| W(f,\tau) \right|^2 df}, \qquad \text{--- (3)}$$

where $f_{BW}(t)$ instantaneous spectral bandwidth. All the above equations with different notations have been adopted from the paper by Barnes (1993) though competing equations for all of them exist in the literature.

For a symmetrical wavelet, like Morlet wavelet (Torrence and Compo, 1998), center frequency of the wavelet at each scale can be assumed to be inversely proportional to the scale. Mathematically, it can be written as,

$$f_C = \frac{k_\psi}{\sigma}, \qquad \text{--- (4)}$$

## Scale Attributes from Continuous Wavelet Transform

where $f_C$, $\sigma$, and $k_\psi$ are center frequency, scale and a wavelet dependent constant respectively. The center frequency can be considered as a representative frequency for a particular scale. Differentiating equation 4, and replacing $f_C$ with $f$, we obtain

$$df = -\frac{k_\psi}{\sigma^2}\, d\sigma \,. \qquad \text{--- (5)}$$

Changing from frequency to scale in the equations 1, 2, and 3 and assuming the wavelet dependent constant $k_\psi = 1$, we obtain

$$\tilde{f}_C(t) = \frac{\int_0^\infty \frac{1}{\sigma^3} |F_W(\sigma,t)|^2 d\sigma}{\int_0^\infty \frac{1}{\sigma^2} |F_W(\sigma,t)|^2 d\sigma}, \qquad \text{--- (6)}$$

$$\tilde{f}_D^2(t) = \frac{\int_0^\infty \frac{1}{\sigma^4} |F_W(\sigma,t)|^2 d\sigma}{\int_0^\infty \frac{1}{\sigma^2} |F_W(\sigma,t)|^2 d\sigma}, \qquad \text{--- (7)}$$

and

$$\tilde{f}_{BW}^2(\tau) = \frac{\int_0^\infty \frac{(1-\sigma \tilde{f}_C)^2}{\sigma^4} |F_W(\sigma,t)|^2 d\sigma}{\int_0^\infty \frac{1}{\sigma^2} |F_W(\sigma,t)|^2 d\sigma}, \qquad \text{--- (8)}$$

where $\tilde{f}_C$, $\tilde{f}_D$, and $\tilde{f}_{BW}$ are instantaneous center frequency, instantaneous dominant frequency and instantaneous bandwidth respectively from the scalogram $F_W(\sigma,t)$. Thus equation 6, 7 and 8 allows us to compute these spectral attributes directly from a scale map.

### Example

Comparisons of instantaneous spectral attributes based on frequency and scale for a seismic trace (Figure 1a) are shown in Figures 1-b, c, and d. We note the remarkable similarity between these attributes although the attributes computed from the scalogram seems to be smoother. This is expected because a scale represents a band of frequency and not a single frequency. Therefore, overlap of frequencies from one scale to another produces smoother attributes, but the overall agreement is excellent. The favorable comparison is a consequence of the scale to center frequency mapping for a Morlet wavelet. Computation of attributes based on scalogram is much faster than those based on TFCWT. Therefore, from practical point of view, attribute definitions based on scale (equations 6, 7, and 8) are computationally efficient at the cost of slight smoother representation of the information. This efficiency translates to saving considerable amount of time depending upon length and number of traces in a large 3D data set.

The next logical step is to compare results on a real seismic section and examine if there are any difference in interpretation. Instantaneous dominant frequency from TFCWT using equation 2 is shown in Figure 2. Instantaneous dominant frequency from scalogram using equation 5 is displayed in Figure 3. In both Figures (2 and 3) the frequency range on the colorbar is kept the same. Figure 2 and 3 indicates remarkable similarity and virtually can be considered as identical. Difference in time-frequency resolution is minimal. It is important to note that the computation of Figure 3 is about 5 times faster than Figure 2. Therefore, scale based attributes are more efficient from practical point of view. Figure 4 is produced using the STFT with a "Hanning" window of 200 ms window length.

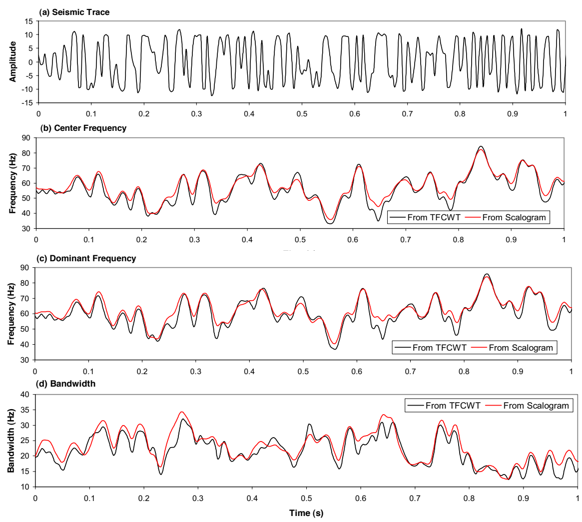


Figure 1: a) A seismic trace of length 1sec sampled at 4ms. It was used to compute frequency and scale based attributes. b) Instantaneous center frequency attribute, c) instantaneous dominant frequency attribute, and d) instantaneous spectral bandwidth computed from TFCWT (shown in black) and from scalogram (shown in red).

# Scale Attributes from Continuous Wavelet Transform

## Conclusions

Scalogram obtained from continuous wavelet transform can provide useful average measures that are directly interpretable in terms of time-frequency attributes. We obtain new expressions for commonly used attributes such as dominant frequency, center frequency and bandwidth directly from a scalogram. These attributes can be efficiently calculated without converting a scalogram into a time-frequency spectrum. Field example presented in this paper shows a factor of 5 computational efficiency using scale based computation. In addition these attributes avoids the subjective choice of a window length commonly used in STFT. This is because the wavelet acts as an effective window via dilatation process.

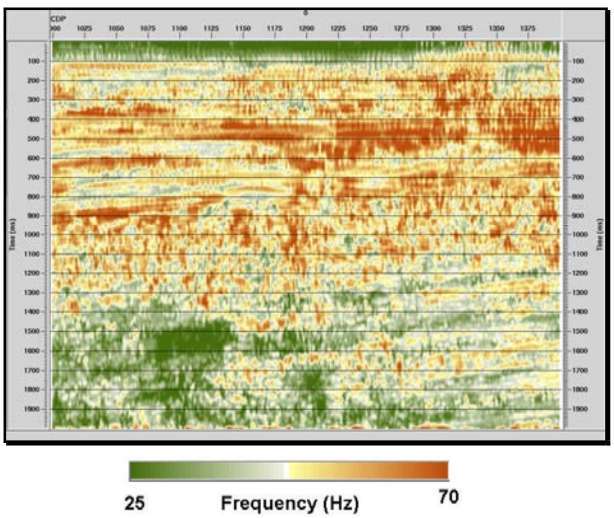


Figure 2: Instantaneous dominant frequency from TFCWT.

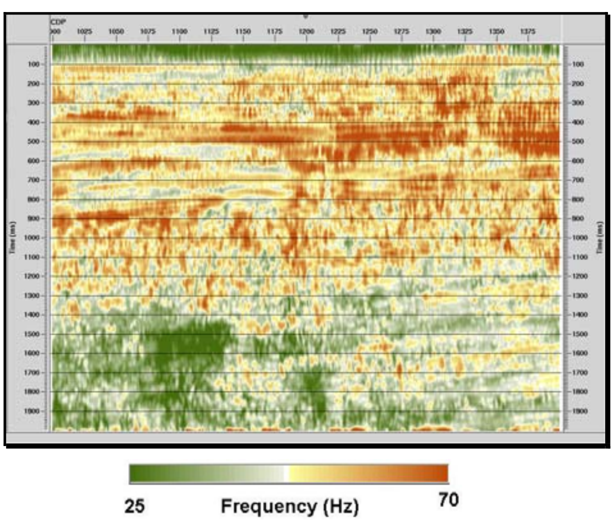


Figure 3: Instantaneous dominant frequency from scalogram.

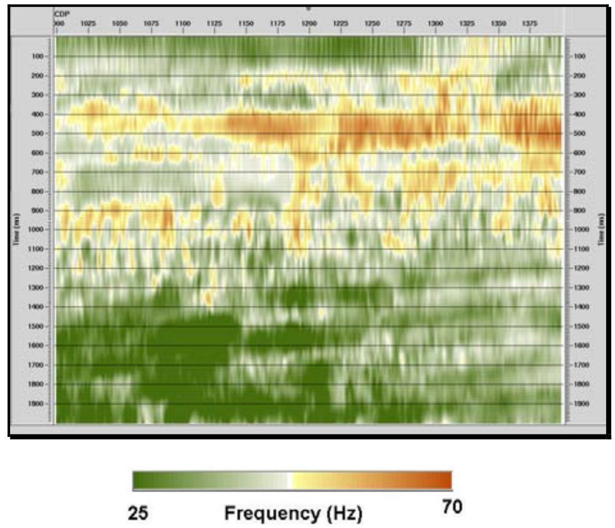


Figure 4: Instantaneous dominant frequency from STFT using 200 ms window length.

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**Scale Attributes from Continuous Wavelet Transform**

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