ground water

Technical Note/

Improved Water Table Dynamics in MODFLOW

by Tom Clemo[1]

Abstract

The standard formulation of a block-centered finite-difference model, such as MODFLOW, uses the center of the cell as the location of a cell node. Simulations of a dynamic water table can be improved if the node of a cell containing the water table is located at the water table rather than at the center of the cell. The LPF package of MOD-FLOW-2000 was changed to position a cell's node at the water table in convertible cells with a water table. Improved accuracy in the upper regions of an unconfined aquifer is demonstrated for pumping from a partially penetrating well. The change introduces a nonlinearity into the solution of the flow equations that results in slightly slower convergence of the flow solution, 7% slower in the presented demonstration. Accuracy of simulations is improved where vertical flow is dominated by a moving water table, but not when a large water table gradient dominates over the water table movement.

Introduction

The MODFLOW series of ground water simulation codes, developed by the U.S. Geological Survey, is possibly the most relied upon set of numerical modeling tools in the hydrogeology community. MODFLOW (McDonald and Harbaugh 1988) is based on a three-dimensional block-centered finite-difference formulation. The finite-differ-ence equations of flow through the subsurface use a single value to describe the hydraulic head within a cell. The equations are formed using the approximation that the cell head value is the head at a specific location, the cell's node. The term "block-centered" indicates that the node is located at the center of the cell. Simulations of fluid flow near a moving water table can be improved if the node of a water table bearing cell is located at the water table rather than at the center of the cell. Dewatering of the aquifer at the water table produces a challenge to the finite-difference formulation. The dynamic movement of the water table changes the region of saturated flow. The change affects the volume of void space available for water storage as well as the physical dimensions of the saturated region. The change in storage is modeled through the specific yield parameter. Specific yield is applied to cells with a water level below the top of the cell. While the change in storage occurs at the water table, the influence is applied to the entire cell. This creates an inaccuracy in the finite-difference approximation that can be reduced by moving the cell node to the water table elevation.

I find the implication of moving the water table is much easier to understand if I change viewpoints from water storage to the addition or removal of water. The change in hydraulic head would be the same if instead of a decrease in void space, an equivalent volume of water was added. The concept is consistent with the phrase "water coming out of storage" that is often used to describe the effects of specific storage. With respect to specific storage, this source of addition or removal of water would be distributed throughout the cell volume. To approximate the source as occurring at the center of the cell is appropriate.

For a change in the water table position, the fluid source should be positioned at the water table since that is where the change in void space occurs. Approximating the source at the center of the cell is equivalent to instantaneous transport of the "added or removed water" from the water table to the center of the cell. The impact on the flow calculations is that the influence of the water table occurs too quickly at depth. In the case of pumping from a well, this impact causes a shift to less drawdown in the delayed yield portion of the drawdown curve. In terms of the program, the change in cell position only changes the vertical cell-to-cell conductance between the cell with the water table and the cell immediately below.

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Calculating Cell-to-Cell Conductance

The MODFLOW-2000 manual (Harbaugh et al. 2000) describes the formulation of the finite-difference approximation used in the MODFLOW codes. Repeating the development of those equations here is not productive. Equation 1 is equation 25 of the MODFLOW-2000 manual with cell dimensions depicted in Figure 1a.

$$CV_{i,j,k+\frac{1}{2}} = \frac{DELR_j DELC_i}{\frac{\frac{1}{2}THICK_{i,j,k}}{VK_{i,j,k}} + \frac{\frac{1}{2}THICK_{K_{i,j,k+1}}}{VK_{i,j,k+1}}} \tag{1}$$

$CV_{i,j,k+1/2}$ is the net vertical conductance resulting from the series summation of the individual conductance of the two cells. $DELR_j$ is the thickness of the jth column and $DELC_i$ is the thickness of the ith row of the model. $THICK_{i,j,k}$ is the vertical thickness of layer k at column j and row i. $VK_{i,j,k}$ is the hydraulic conductivity of the cell.

Figure 1b is a cross sectional view of two cells with elevations identified. $BOTM_{i,j,k}$ is the bottom of the cell i, j, k. $BOTM_{i,j,k-1}$ is the bottom of the cell above. $BOTM_{i,j,0}$ is the top of the model. $HNEW_{i,j,k}$ is the hydraulic head in the cell and is located in the figure at the elevation of the same value. In MODFLOW, the hydraulic head and physical elevations are referenced to the same datum. Therefore, if $HNEW_{i,j,k} < BOTM_{i,j,k-1}$, then a water table exists in the cell at the elevation $HNEW_{i,j,k}$. The elevation of the center of the cell is labeled $CTR_{i,j,k}$. $\mathrm{CTR}_{i,j,k} = 1/2(BOTM_{i,j,k-1} + BOTM_{i,j,k})$. CTR is introduced here for convenience. It is not a MODFLOW variable. If $HNEW_{i,j,k} < BOTM_{i,j,k-1}$, then $CTR_{i,j,k} = 1/2(HNEW_{i,j,k} + BOTM_{i,j,k})$. Replacing $1/2THICK_{i,j,k}$ in Equation 1,

$$CV_{i,j,k+\frac{1}{2}} = \frac{DELR_j DELC_i}{\frac{\frac{1}{2}(HNEW_{i,j,k} - BOTM_{i,j,k})}{VK_{i,j,k}} + \frac{CTR_{i,j,k+1} - BOTM_{i,j,k+1}}{VK_{i,j,k+1}}} \tag{2}$$

Changing the position of the node location to the water table results in

$$CV'_{i,j,k+\frac{1}{2}} = \frac{DELR_j DELC_i}{\frac{HNEW_{i,j,k} - BOTM_{i,j,k}}{VK_{i,j,k}} + \frac{CTR_{i,j,k} - BOTM_{i,j,k+1}}{VK_{i,j,k+1}}} \tag{3}$$

Note the elimination of the $\frac{1}{2}$ in the first term of the denominator and the prime to indicate the modified CV. The difference between the two conductances approaches zero as the water table approaches the bottom of the cell.

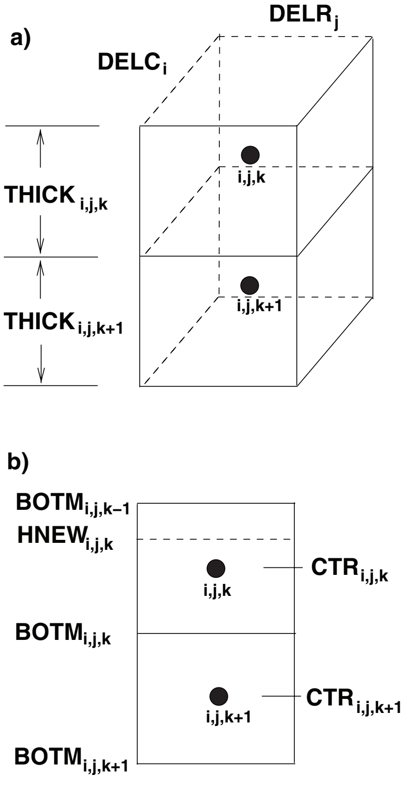

Figure 1. Two finite-difference model cells depicting (a) the variables used in Equation 1 (after Harbaugh et al. 2000), and (b) cell bottom and water table elevations used in Equation 2.

Implementation

Only a small change is needed to implement a water-table-following node in the layer-property flow package. In the subroutine SGWF1LPF1VCOND (gwf1lpf1.f, dated 13JAN2000), the variable BOVK1 represents $\frac{\frac{1}{2}THICK_{i,j,k}}{VK_{i,j,k}}$, HYC1 has been set to $\mathrm{VK}_{i,j,k}$, and HALF is 0.5. The following code is used.

```
C4—CALCULATE INVERSE LEAKANCE FOR
  CELL.
BBOT = BOTM(J,I,LBOTM(K))
TTOP = BOTM(J,I,LBOTM(K)–1)
IF (LAYTYP(K).NE.0) THEN
   HHD = HNEW(J,I,K)
   IF(HHD.LT.TTOP) THEN
      TTOP = HHD
      FAC = 1.
   ELSE
      FAC = HALF
   ENDIF
ELSE
   FAC = HALF
ENDIF
BOVK1 = (TTOP-BBOT)*FAC/HYC1
```

Comparison to WTAQ

The influence of the position of the water table node using MODFLOW may be compared to WTAQ3 (Barlow and Moench 1999) calculations for a hypothetical homogeneous aquifer. WTAQ3 provides a numerical implementation of analytic solutions (Moench 1997) to homogeneous water table aquifer responses to pumping from a partially penetrating finite-diameter well. The properties of the hypothetical aquifer used for the comparison are listed in Table 1.

The pumping well has a radius of 0.0508 m and is screened over an interval of 8 to 12 m below the water

Table 1
Aquifer Characteristics Used in the Comparison

Parameter	Value
Horizontal K	0.0001 m/s
Vertical K	0.0001 m/s
Specific storage	0.0001 per m
specific yield	0.38
Aquifer thickness	18 m

table. Two MODFLOW model definitions are used in the comparison. Both have the same 43 rows and 45 columns. The horizontal grid spacing was constructed such that nodes occurred close to the following distances from the pumping well: 3.52, 7.08, and 22.1 m. One model has 21 layers, the other 26. Each layer in the 21-layer model is 1 m thick, except for 0.5 m thick layers above and below the pumping interval and at the top and bottom of the model. The 26-layer model has a fine grid near the water table. In the 26-layer model, the uppermost layer is 0.1 m thick and there are eight layers in the upper 3 m. In both cases the water table is initially at the top of the upper layer.

Figure 2 presents calculations of drawdown at distances from the pumping well of 3.52, 7.08, and 22.1 m at depths of 1, 6, and 10 m below the water table. Four sets of data are presented in each panel. All of the MODFLOW simulations are similar to the WTAQ calculations. The largest deviation between the calculations occurs at nodes 1m from the water table and at times when the drawdown is first impacted by the water table decline causing a brief plateau, or delayed yield period, in the drawdown. After the plateau region, the impact of water table decline is fast with respect to the time scale and the drawdown increases rapidly again.

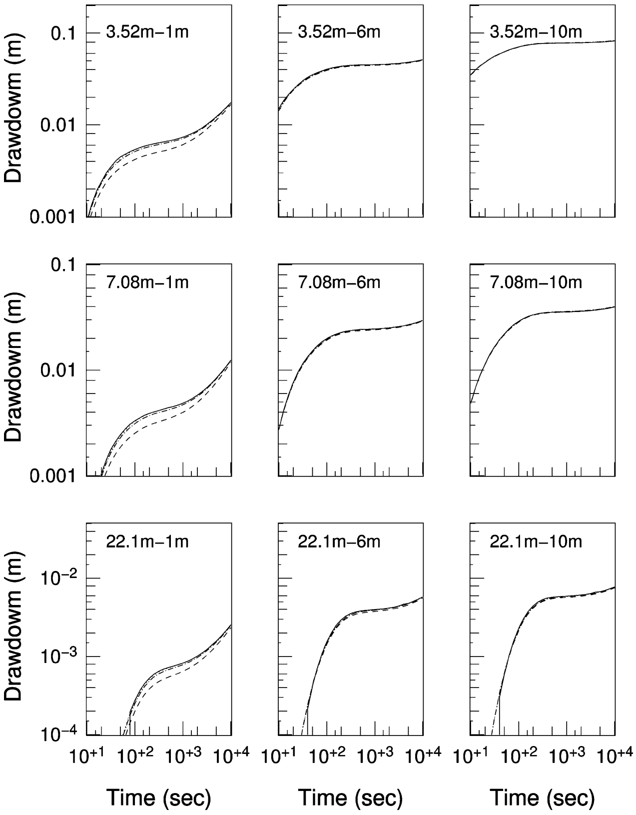

Figure 2. Comparison of drawdown at various locations. WTAQ calculation (solid), unmodified MODFLOW with 21 layers (dashed), unmodified MODFLOW with 26 layers (dotdash), and modified MODFLOW with 21 layers (dotted). The dotted line overlays the solid line at these scales.

Figure 3 shows the region of greatest disagreement: the plateau region of the location nearest to the water table, 1 m depth, and closest to the well, 3.52 m. In this figure, the differences between model calculations are more clearly presented. The dot-dash line presents head calculations at this location from the 26-layer model. These are a definite improvement to head calculations at this location from the 21-layer model (dashed line). The dotted line of head calculations from the modified MODFLOW code fails to overlay the WTAQ calculations (solid line) only at very early times.

At a given level of discretization, the modified code provides a much more accurate representation of water table dynamics over the original MODFLOW code. The accuracy of the original code is improved with finer discretization, but at the cost of larger and slower simulations. Simulating the change in conductance from water table movement also slows the simulation, but not as much and with greater improvement in accuracy. For the 21-layer geometry, the water table modification slows the execution speed of the model by 7%. Using the unmodified code with 26 layers instead of 21 layers requires 31% more time to reach a solution. This improvement in accuracy applies to the dynamics of the simulation.

Conclusion

A theoretical argument has been presented to suggest that a more accurate simulation of water table dynamics

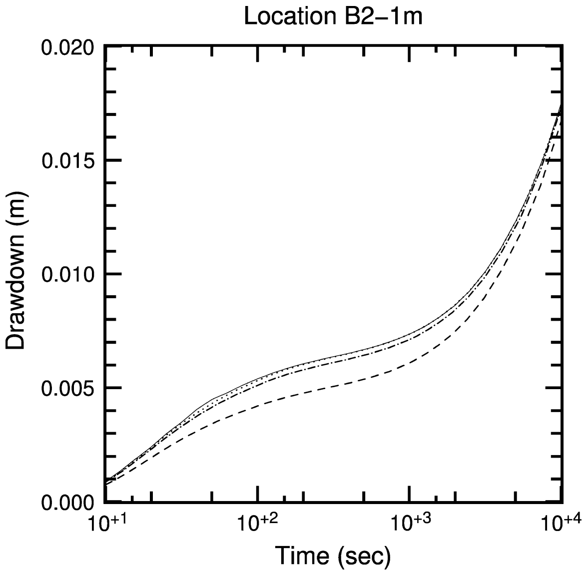

Figure 3. Semi-log comparison of drawdown at a depth of 1 m below the water table 3.52 m away from the well. WTAQ (solid), unmodified with 21 layers (dashed), unmodified with 26 layers (dot-dash), and modified with 21 layers (dotted).

with block-centered finite difference simulation of fluid flow can be accomplished if the vertical positions of the nodes dynamically follow the water table. The USGS MODFLOW-2000 code was so modified. Comparisons of drawdown simulations from both the modified and unmodified codes with the analytically based WTAQ code reveal: (1) fine vertical discretization of layering near the watertable improves the accuracy of the MODFLOW calculations but at a cost of longer execution times; and (2) dynamically following the water table is more accurate than the fine vertical discretization and at less cost in terms of execution times. The emphasis here has been on the dynamic behavior. The modification can have a detrimental effect on the accuracy of steady state flow simulations under conditions of a sloping water table.

A small modification to the MODFLOW-2000 code is needed to implement water table following. This modification for the layer-property flow package is given in this paper.

Acknowledgments

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