Traveltime tomography of crosshole radar data without ray tracing

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**ABSTRACT**

We presented a least-squares traveltime inversion algorithm for crosshole ground-penetrating radar (GPR) direct-arrival data. The proposed scheme used the eikonal equation as traveltime functional and thus avoided tracing rays during inversion. The Jacobian matrix is constructed by a finite-difference approximation via the perturbation of slowness. The solutions were obtained by an iteratively linearized inversion approach. A smoothness-type regularization was implemented to stabilize the solutions. Traveltime calculations in forward modeling were performed by a finite-difference eikonal solver that allows modeling wavefronts. Matrix inversions were achieved by using conjugate gradient least-squares (CGLS) and LSQR algorithms. Broyden’s method was used to accelerate the calculation of the Jacobian matrix when the number of model parameters was large. We tested the proposed method on three synthetic data sets and on a field data set from the Boise Hydrogeophysical Research Site (BHRS), Idaho; and we compared our model for the field data with the one obtained by a ray-tracing-based algorithm. This comparison indicated that the suggested inversion scheme was able to generate a solution as good as the one resulting from a conventional ray-based scheme. The synthetic data were obtained from simple to complex subsurface velocity distributions, including low- and high-velocity anomalies. Additionally, an image quality analysis was performed by calculating model covariance and model resolution matrices for one of the synthetic models having a complex subsurface structure and for the model resulting from the field data. All inversions were characterized by fast and stable convergence. Tests with noisy data sets indicated that the tomograms were relatively insensitive to noise in the data. It was also observed that the LSQR algorithm produced better results than the CGLS did in the tests with the synthetic models having complex subsurface structures. We considered the proposed technique to be an efficient traveltime inversion scheme for crosshole radar data.

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1. Introduction

Ground-penetrating radar (GPR) is a non-invasive technique for the investigation of the electrical properties of the subsurface. It is applied to a wide range of shallow geophysical investigation tasks. This includes sedimentology and hydrogeology (Fisher et al., 1992; van Overmeeren, 1998; Neal, 2004); environmental and engineering studies (Knight, 2001; Annan, 2004; Porsani et al., 2004); shallow aquifer investigations (Cardimona et al., 1998); cavity detection (Zhou and Sato, 2004); soil moisture measurements and porosity estimations (Greaves et al., 1996; Huisman et al., 2003; Hanaly and Al Hagrey, 2006; Turesson, 2006; Strobbia and Cassiani, 2007); archaeological prospections (Neubauer et al., 2002; Conyers, 2006; Şeren et al., 2008); and glaciology and permafrost studies (Arcone et al., 1998; Hinkel et al., 2001; Eisen et al., 2003).

In surface GPR surveys, common-offset reflection profiling is usually preferred to determine subsurface structure. However, resolution of a surface GPR image quickly decreases with depth. Therefore, subsurface radar velocity characterization at the deeper parts of the section may not be accurate enough for reliable interpretations. As for crosshole seismic tomography (Bregman et al., 1989; Aldridge and Oldenburg, 1993; Lehmann, 2007), a GPR survey based on crosshole configuration may be better suited if a detailed velocity distribution at greater depth is required. Recently, there are a number of studies on crosshole GPR tomography (Clement and Knoll, 2000; Holliger et al., 2001; Tronicke et al., 2001; Zhou et al., 2001; Musil et al., 2003; Gloaguen et al., 2005; Clement and Barrash, 2006; Ernst et al., 2007a,b; Giroux et al., 2007; Irving et al., 2007), which indicate the wide-spread interest in the GPR method.

In crosshole radar tomography, traveltime inversion of the transmitted data is used for the estimation of radar velocities. In general, this technique requires a ray-tracing algorithm for the computation of traveltimes. Thus ray tracing is an essential part of the traveltime tomography for both accurate calculation of traveltimes and construction of a matrix consisting of the lengths of the raypaths in each slowness cell. Either straight or curved ray approach is implemented according to the assumptions on the velocity distribution. Straight rays may be adequate if a medium is characterized by smooth and negligibly small velocity variations. In case of strong variations, using curved rays is necessary to obtain accurate
results (Aldridge and Oldenburg, 1993; Clement and Knoll, 2000; Lehmann, 2007). On the other hand, Ammon and Vidale (1993) introduced an algorithm for tomographic inversion of seismic first-arrival times without performing ray tracing. Their algorithm is based on finite-difference solution of the eikonal equation for traveltimes. In other words, traveltimes are calculated by tracing wavefronts instead of rays (Vidale, 1988). In their scheme, velocity estimation is carried out by an iteratively linearized inversion, in which the linearization is achieved by a Taylor series expansion. The Jacobian (sensitivity) matrix, which consists of the partial derivatives of the traveltimes with respect to the model parameters, is calculated by a finite-difference approximation based on the perturbation of the cell slowness.

In the present study, we applied Ammon and Vidale’s (1993) approach to the traveltime inversion of crosshole radar direct-arrival times for the velocity estimation. In the proposed method, construction of the sensitivity matrix is based on the functional description of traveltime calculations, which means a non-linear function (i.e., eikonal equation) providing traveltimes for a given slowness model. Smoothness-type regularization is used to stabilize the solution. Matrix inversion is implemented by using the conjugate gradient least-squares (CGLS) (Hestenes and Stiefel, 1952; Scales, 1987; Scales et al., 2001) and the LSQR (Paige and Saunders, 1982) algorithms. Broyden’s (1965) method was performed to expedite the calculation of the Jacobian matrix when a model was composed of a large number of parameters. We tested the suggested method by inverting both synthetic and field data sets. The synthetic data were based on three subsurface models having low- and high-velocity contrasts. The synthetic first-arrival traveltimes were obtained via traveltime picking from the waveform data calculated by a finite-difference time-domain (FDTD) scheme (Irving and Knight, 2006) over a gridded velocity field. A crosshole radar traveltime data set from the Boise Hydrogeophysical Research Site (BHRS), Idaho (Courtesy of Center for the Geophysical Investigation of the Subshallow Surficial System (CGISS) at Boise State University) was used for the tests with field data. Additionally, we tested the effects of noise on the solutions of the inverse problem by adding Gaussian random errors to the synthetic traveltimes. Furthermore, we showed how Broyden’s method could improve computational efficiency by reducing the calculation time required for the Jacobian matrix. To quantity the quality of the solutions we calculated model covariance and model resolution matrices for the synthetic model representing the most complex subsurface structure and for the model obtained from the field data.

2. Methodology

2.1. Formulation of the inverse problem

In the inversion scheme, the following total objective function is minimized for the estimation of the radar velocities

$$ \Phi(\Delta s) = \Phi_d(\Delta s) + \lambda^2 \Phi_m(\Delta s) = \| \sqrt{W_d(F(\Delta s - \Delta d))} \|^2 + \lambda^2 \| W_m \Delta s \|^2. \quad (1) $$

where $\Phi_d$ and $\Phi_m$ are the data misfit and the model norm, respectively. $\lambda$ is a weighting factor for the trade-off between data misfit and model smoothness, $W_d$ is the data weighting matrix, $F$ is the Jacobian matrix consisting of the partial derivatives, $\Delta s$ is the slowness adjustment vector, $\Delta d$ is the traveltime residual vector and $W_m$ is the weighting matrix for the model parameters. Minimization of $\Phi(\Delta s)$ leads to the following normal equations

$$ (j^T W_d F + \lambda^2 W_m^T W_m) \Delta s = j^T W_d \Delta d. \quad (2) $$

The solution of Eq. (2) for $\Delta s$ requires explicit matrix–matrix multiplications, which are computationally expensive. Therefore, it is more efficient to find the least-squares solution of the following system whose solution is equivalent to that of Eq. (2) (Johnson et al., 2007; Greenhalgh et al., 2006),

$$ \sqrt{W_d^T W_d} \Delta s = \begin{bmatrix} \sqrt{W_d^T W_d} \\ 0 \end{bmatrix}. \quad (3) $$

Considering large and sparse matrices resulting from a tomography application, a solution for Eq. (3) may be obtained by an iterative approach using either the CGLS or LSQR algorithms, which are analytically equivalent. The LSQR algorithm is based on the Golub–Kahan bidiagonalization (Golub and Kahan, 1965) and is more reliable in case of moderate to severe ill-conditioned problems (Paige and Saunders, 1982). In the present study, both the CGLS and the LSQR algorithms were tested and the sparsity of the matrices was taken into account by implementing the full-index scheme given by Scales (1987). The data weighting matrix $W_d$ was assumed being the identity matrix, which implied that the data were uniformly weighted. A Laplacian smoothing, which is based on the Laplacian difference (Ammon and Vidale, 1993; Alumbaugh and Newman, 2000) of the slowness model, was used for the parameter weighting matrix $W_m$.

2.2. Finite-difference calculation of the Jacobian matrix

For a tomographic inversion problem consisting of $m$ observations and $n$ model parameters with constant slowness cells, the elements of the $m \times n$ Jacobian matrix are given by

$$ J_{ij} = \frac{\partial F_i}{\partial s_j}, \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n. \quad (4) $$

Here, $s$ represents the slowness of an individual cell and $F$ is traveltime functional. Following Ammon and Vidale (1993), we constructed the Jacobian matrix by a numerical differentiation. This approach is based on the perturbation of the slowness model cell by cell and the calculation of the partial derivatives by a finite-difference approximation (Lines and Treitel, 1984). Using the forward-difference formula, Eq. (4) can be approximated by

$$ \frac{\partial F_i}{\partial s_j} \approx \frac{F_i(s_j + \delta s_j) - F_i(s_j)}{\delta s_j}. \quad (5) $$

where $\delta s$ is the amount of the perturbation. Formally, Eq. (5) requires two forward calculations for each $s_j$. In practice, only one forward calculation $F(s_j + \delta s_j)$ is needed to construct the partial derivative matrix; the reason being traveltime calculation $F(s_j)$ is already available from the traveltime residual calculations in the previous iteration. Once the forward calculation for the perturbed model is completed, the $j$th column of the Jacobian matrix can easily be obtained using Eq. (3). We performed some tests studies (not shown here) to determine the best $\delta s$. For this reason, the synthetic data set generated from Model 1, which has the highest velocity contrast (Fig. 1a) among the synthetic models considered in the present study, was inverted with a variety of $\delta s$ including $+0.1\%$, $+0.5\%$, $+1.0\%$ and $+5.0\%$. The tests with $0.5\%$ and $1.0\%$ generated very similar results and they were the most successful solutions in terms of inverted velocity distributions and traveltime residuals. Since the solution with $0.5\%$ yielded a velocity reconstruction having minimum traveltime residual, we carried out all inversions in this study with that $\delta s$. Therefore, we perturbed the slowness of each individual cell in the model by $+0.5\%$. 

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2.3. L-curve analysis for the parameter $\lambda$

The parameter $\lambda$ in Eq. (1) controls the amount of smoothness introduced. The L-curve approach (e.g., Hansen and O’Leary, 1993) can be used to obtain an optimal value for $\lambda$. This method is based on the calculation of the data misfit ($||s - d||^2$) and the model norm (model roughness) ($||Wm^s||^2$) for a range of $\lambda$ values. The plot of the data norm versus the model norm typically yields an L-shaped curve. The $\lambda$ value at the corner or at the point of maximum curvature is the optimal choice providing low values for both the data and the model norms. This method is particularly useful in case of an unknown amount of noise in the data (Moret et al., 2006; Johnson et al., 2007). We carried out an L-curve analysis on both noise-free and noisy synthetic data obtained from Model 1 (Fig. 1a). The noisy data set was obtained by adding zero-mean Gaussian random noise with a standard deviation of ±0.5 ns to each traveltime in the data set. A 5-iteration inversion was performed for each $\lambda$ value ranging from 2 to 20. The resulting L-curves are shown in Fig. 2. Since the curves are not characterized by a sharp change in its curvature, a broad range of $\lambda$ values (e.g., $\lambda = 4-8$) may be suitable for the inversion, and will eventually yield similar models.

2.4. Broyden’s update scheme for the Jacobian matrix

The least-squares solution of Eq. (3) requires the partial derivative matrix $J$ to be recalculated at each iteration. Construction of this matrix by the explicit calculation of the partial derivatives using Eq. (5) is a computationally expensive and time consuming task. This can be avoided if the matrix $J$ is replaced by the numerical scheme proposed by Broyden (1965). After computing the Jacobian matrix for the initial model it can be approximated in all iterations by the updating scheme and thus avoiding the use of Eq. (5). Among the other updating schemes, Broyden’s method is commonly preferred for arriving at the solution of a number of non-linear problems due to its stability and rate of convergence (Loke and Barker, 1996). Broyden (1965) suggested the following updating equations for the partial derivative matrix $J$ at the $(k+1)$th iteration

$$J^{k+1} = J^k + \frac{\Delta F^k - f^k\Delta s^k}{||\Delta s^k||^2} \Delta s^k,$$

where

$$\Delta F^k = F(s^{k+1}) - F(s^k),$$

$$\Delta s^k = s^{k+1} - s^k.$$

2.5. Verification of the inverted velocity models

For verifying the velocity reconstructions for Model 3 (Fig. 1c) and the model resulting from BHRS field data, we calculated model Fig. 1. Models used to generate synthetic data sets. (a) Model 1 (Ammon and Vidale, 1993) (b) Model 2 (Irving et al., 2007) (c) Model 3 (Irving and Knight, 2006).

Fig. 2. L-curve plots for the inversion of the synthetic data obtained for Model 1. (a) Noise-free data (b) noisy data. Plots for $\lambda$ values between 2 and 20.
covariance (MCM) and model resolution matrices (MRM), which provide quantitative measures of the quality of the inversion results (Menke, 1984; Tarantola, 2005). The main diagonal of the MCM indicates how noise in the data and errors from inappropriate model assumptions are related to uncertainties in the model parameter estimation (Alumbaugh and Newman, 2000; Moret et al., 2006). For a linearized inversion scheme, the MCM can be calculated as follows (Alumbaugh and Newman, 2000)

\[
MCM = \left( f' \right)^T W_d f' + \lambda^2 W_m^T W_m \right)^{-1},
\]

where \( f' \) is the Jacobian matrix of the final iteration.

The MRM indicates how well each model parameter is resolved. In an ideal case, it should be an identity matrix (Menke, 1984). Unlike for linear problems, the MRM for a linearized scheme must be calculated.

Table 1

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Table 2

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after the inversion and is obtained by (Alumbaugh and Newman, 2000)

$$MRM = \left((f^T)^T W_d f^T + \lambda^2 W_m W_m^T \right)^{-1} \left((f^T)^T W_d f^T \right).$$  

(8)

Diagonal values less than one imply that the slowness of the considered cell is influenced by the slowness of the adjacent cells (Alumbaugh and Newman, 2000; Moret et al., 2006).

To obtain an image of the model resolution, we plotted the diagonal entries of the MRM. Since the inversion problem in this study is regularized by a Laplacian smoothing operator, the model parameters cannot be resolved uniquely. Instead, they are affected by an averaging process including the surrounding cells. The diagonal elements in the MRM are thus less than one. Since the elements on the main diagonal of the MCM are the variances of each parameter in the slowness model, we calculated uncertainties for the parameters in radar velocities by using the following equation introduced by Moret et al. (2006)

$$u_j = \frac{0.5}{s_j + \sqrt{MCM_{jj}}} - \frac{0.5}{s_j - \sqrt{MCM_{jj}}}.$$  

(9)

We also obtained a ray coverage plot for Model 3 and the model from BHRS field data by performing a post-inversion ray tracing on the final velocity model. The plot is useful to assess the quality of resulting images because the velocities are in general more reliable if the region is characterized by many crossing rays (Moret et al., 2006). After calculating the traveltimes for a certain transmitter–receiver configuration by the eikonal solver, the ray path having the minimum traveltme can be easily obtained by following the steepest gradient direction from the receiver to the transmitter (Vidale, 1988).

3. Examples

3.1. Models and synthetic data

We tested the scheme by using three different synthetic traveltimes data sets. All models used for generating the data sets are shown in Fig. 1. Model 1, which is similar to the model used by Ammon and Vidale (1993), includes a 20% high- and a 20% low-velocity block embedded in a homogeneous background (Fig. 1a). Both transmitters and receivers were located at every 0.25 m between 0.63 and 10.38 m, which was 0.5 m away from the left and right model edges. Model 2 was adapted from Irving et al., 2007 (Fig. 1b). The authors used this model to simulate saturated zones. It has a number of blocks with various shapes and velocities ranging from 6% higher to 5% lower velocities than that used for the background. Similar to their configuration, we placed transmitters and receivers at 0.25 m intervals between 0.5 and 11.5 m, which was 0.5 m away from the left and right model edges. Model 3 was adopted from Irving and Knight’s (2006) crosshole GPR simulation study. Their model is composed of radar velocities ranging from 0.053 to 0.067 m/ns (Fig. 1c). We illustrated the same velocity structure over a deeper and wider subsurface model than theirs. The transmitters and the receivers were located at 0.5 and 0.25 m intervals between 0.63 and 15.63 m, respectively. They were again 0.5 m away from the left and right model edges. This model again represents a saturated zone, but it is more realistic in terms of geology compared to the previous models. Nevertheless, we think that the block structures in Model 1 and 2 may allow us to examine the resolution capabilities of our algorithm.

Synthetic data sets were generated by simulating a multi-offset transillumination survey, where only the direct arrivals were considered. Therefore, the arrivals related to the head waves caused by the air–subsurface interface were not accounted for. The synthetic first-arrival traveltimes were obtained by traveltime picking from the waveforms calculated by a two-dimensional FDTD scheme (Irving and Knight, 2006) over a gridded velocity field. A 100-MHz Blackman–Harris pulse was used as source. Fig. 3 shows some of the gathers generated from Model 1 and 3. The picked first-arrival times are also superimposed on the gathers. Table 1 lists the models and the corresponding modeling parameters used for synthetic data generation.

3.2. Test studies with synthetic data

A homogenous velocity distribution was used for the initial model during inversion studies. To determine the velocity of the starting
model, a velocity value was estimated for each source–receiver pair by dividing the source–receiver distance by the corresponding synthetic traveltime and then having an average of these values (Jackson and Tweeten, 1994). A fixed iteration number was used for inversion tests. The inversion process was stopped after 10 iterations for the tests with Model 1 and 2, and after 15 iterations for those with Model 3. Table 2 shows the main parameters used for the inversions of the synthetic data sets.

Accurate calculation of the first-arrival traveltimes is very important in the inversion of traveltime data. This is traditionally done with a ray-tracing algorithm. Alternatively, one could use a numerical representation of the eikonal equation (e.g., Vidale, 1988, 1990). In Vidale’s approach, he uses a finite-difference extrapolation of the wavefronts from point to point on either a 2D or 3D grid. However, in the present work, we use a different numerical scheme to compute first-arrival times first introduced by Podvin and Lecomte (1991). Their algorithm uses Huygens’ principle in a finite-difference scheme based on staggered grid. This algorithm yields accurate traveltimes even if arbitrarily shaped and high-contrast velocity distribution is present. In order to obtain accurate traveltimes a mesh with constant grid interval was employed for forward modeling and it was much finer than the one used for the slowness cells. For Models 1 and 2, we employed 100 grid points per cell and for Model 3 25 grid points per cell. Due to a relatively large number of cells in Model 3 we calculated the traveltimes with a relatively smaller number of the nodes per cell to reduce computational costs. The traveltime residual $r_k$ was calculated at the end of each iteration, using the following equation:

$$r_k = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (t^o_i - t^c_i)^2},$$  \hspace{1cm} (10)

where $M$ is the number of traveltimes, $t^o$ and $t^c$ are the observed (i.e., synthetic traveltimes) and calculated traveltimes, respectively. $i$ denotes observation, and $k$ denotes the iteration number.

3.2.1. Model 1

Fig. 4 displays the reconstructions of Model 1 with different $\lambda$ values of 2, 5 and 10. The solutions were obtained by the CGLS algorithm using homogeneous initial models with a velocity of 0.1 m/ns. The inversion process stopped at the end of 10 iterations. The corresponding residuals are plotted on the velocity tomograms. The locations of the velocity anomalies are also shown on each tomogram and indicate a good match between the true anomaly locations and their tomographic reconstructions. The effects of the weighting factor $\lambda$ are clearly visible: increasing this factor causes smoother models with relatively higher rms errors. The residuals as function of iteration numbers for $\lambda$ values of 2, 5 and 10 are shown in Fig. 5. This plot displays a fast decrease in the residuals after a few iterations indicating a quick and stable convergence.

![Fig. 6. Reconstructions of Model 1 using Broyden’s method for $\lambda$ values of 2, 5 and 10. Broyden’s approximation was applied after the third and for all subsequent iterations.](image)

![Fig. 7. Velocity reconstructions of Model 1 with noisy data for $\lambda$ values of 2, 5 and 10.](image)
Fig. 6 shows the solutions with Broyden’s method for λ values of 2, 5 and 10. They were obtained using the CGLS approach with a homogeneous initial model (0.1 m/ns). The Jacobian matrix was replaced by its numerical approximation after the third iteration and for all subsequent iterations. Broyden’s update after the third iteration is based on the test studies that we performed to determine the optimum iteration number before implementing Broyden’s method and found that at least two iterations were required before using Broyden’s scheme (see Fig. 11 for the results of such a test study). The resulting velocity reconstructions are very similar to those shown in Fig. 4. We also investigated the effect of noise on the velocity reconstruction by using the noisy data set generated from Model 1 for L-curve analysis. The inversions were performed by the CGLS approach without using Broyden’s update scheme. The inversion process was started based on a homogeneous velocity model (0.1 m/ns) and stopped after 10 iterations. Fig. 7 shows the results for the same λ values as those previously used for the noise-free data set inversion. As might be expected, the velocity images were obtained by larger traveltime residuals. Each solution displays an rms error of about 0.5 ns, indicating the standard deviation of the noise which was added to the data. As seen from the figure, the velocity anomalies are better visible on the images obtained by the larger weighting factor (λ = 5 and 10) due to the noise in the data.

3.2.2. Model 2
This test represents a more complex blocky subsurface with more varying low- and high-velocity anomalies compared to Model 1. The synthetic data for this model were inverted by both the CGLS (Fig. 8a–b) and the LSQR algorithms (Fig. 8c–d) for the λ values of 2 and 6 and Broyden’s update scheme was not applied during these inversions. Each test was stopped at the end of 10 iterations. A homogeneous velocity of 0.06 m/ns was used as starting model for each inversion. The true locations of the velocity anomalies are again superimposed on each final tomogram. The reconstructions indicated almost the same traveltime residuals (e.g., 0.079 ns for Fig. 8a and 0.081 ns for Fig. 8c) for the solutions with the same λ value once the inversion converged. Overall, there is a good match between true velocity anomalies and those obtained by the inversions. Increased traveltime residuals and some smearing at the velocity anomalies are present on the velocity images with increasing λ. A smoothing effect can be observed particularly in the high-velocity anomaly at 10 m. The results based on the LSQR method display relatively lesser smoothing with increasing λ value.

3.2.3. Model 3
Fig. 9 shows the results of the inversion with the synthetic data from Model 3. The radar velocity tomograms were calculated by the LSQR algorithm with λ values of 2, 4 and 6 without employing Broyden’s update method. A constant velocity distribution of 0.061 m/ns, estimated from the synthetic traveltime data, was used as the initial model. As seen from the figure, each tomogram clearly displays velocity changes in the model (Fig. 9a). In this test, based on noise-free data, the inversion with a λ value of 2 yields the lowest rms error (0.14 ns) at the end of the 15th iteration (Fig. 9b). The smoothing effects introduced by higher λ values can easily be observed in the corresponding images together with larger rms errors (Fig. 9c, d). The change of traveltime residuals with respect to iteration numbers are given in Fig. 10.

Fig. 11 shows a comparison between the final velocity tomograms at the end of 15 iterations. They are based on a λ value of 4 without Broyden’s update (Fig. 11a) and with Broyden’s update after the first, second and third iteration (Fig. 11b–d). According to the figure, the solution using Broyden’s update starting after the first iteration did not yield satisfactory results while the others show consistent results with similar rms errors. The velocity tomogram in Fig. 11b was obtained with larger traveltime residual of 2.02 ns and it is characterized by some distortions in the velocity image. Using Broyden’s update scheme for the Jacobian matrix considerably reduces the required computer time to reach convergence. The solution without Broyden’s method took 82 min while those with Broyden’s update after the first, second and third iterations took 6.8, 11.6 and 17 min as computer time, respectively on a 1.8-GHz IBM PC compatible microcomputer with a memory of 2 GB. Considering both the computer time performances and resulting radar velocity images, the use of Broyden’s update, after the second iteration seems to be the best choice.

Fig. 12 shows the results of the solution appraisal analysis for the inversion with the λ value of 4. Both the noise-free and noisy data sets were considered in this analysis. Noise contamination was obtained by adding Gaussian random variate with a mean of zero and a standard deviation of ±1.5 ns to each traveltime. Broyden’s update method was not used during the solutions of the inverse problem. The inversion processes stopped after 15 iterations. A homogeneous velocity distribution (0.06 m/ns) was used as initial model and the tomograms were calculated based on the LSQR approach. Even though
some discrepancies are present in the velocity image resulting from the noisy data set, each tomogram shows similar velocity distributions (Fig. 12a, e). The traveltime residual of 1.52 ns in Fig. 12e is in good agreement with the maximum noise in the data. The estimated uncertainties are less than 0.001 m/ns in each solution (Fig. 12b, f). This implies that the resulting velocity tomograms are relatively insensitive to the noise in the data and the errors due to the inappropriate model assumptions (i.e., the assumption of smoothly varying model velocities). Both high values of the uncertainties and low values of the model resolution are observed at top and bottom parts of the imaged regions (Fig. 12b, c, f, and g). These parts of the images do also coincide with the unconstrained parts in the ray coverage plots (Fig. 12d, h). For the visibility of the ray coverage plot, we plotted raypaths for every second transmitter–receiver configuration. There is also a good match between the raypath pattern in the ray coverage plot indicating the constrained parts of the image and the relatively high values of the model resolution (Fig. 12c, d, g and h). The same data sets were also inverted by using the CGLS algorithm without Broyden’s update. Fig. 13 shows the resulting velocity images with the same λ and starting model as used in the inversion producing Fig. 12. Surprisingly, the tomograms are clearly different from those obtained with the LSQR approach. Very strong differences are present in the images although they have reasonable traveltime residuals. Compared with the tomograms in Fig. 12, it is obvious that the results with CGLS are not satisfactory. Even though the inversion of the noise-free data set was able to moderately reveal some velocity anomalies in the model, the test with the noisy data set failed to image the velocity distribution. As might be expected, the inverse problem based on Model 3, which is the most complex synthetic model, is more ill-conditioned compared to the other problems considered in this study. When a problem is ill-conditioned, LSQR is more reliable and generates more accurate solutions compared to CGLS (Paige and Saunders, 1982). Therefore, the inversion using LSQR algorithm better imaged the velocity distributions resulting from both noise-free and noisy data sets.

### 3.3. Test studies with BHRS field data

We also tested the algorithm by using a first-arrival traveltimes data set obtained from a crosshole radar experiment at BHRS near Boise, Idaho. It is a wellfield for hydrogeophysical research on heterogeneous alluvial aquifers. The site is characterized by a shallow and unconfined aquifer and includes 18 boreholes drilled through a gravel bar in the vicinity of the Boise River. The sedimentary fill, which is approximately 18 m thick, consists of unconsolidated coarse fluvial deposits including gravels and cobbles with sand lenses (Barrash and Clemo, 2002; Barrash and Reboulet, 2004).

Fig. 14 shows the results of the inversions with the λ values of 4 and 6 together with the solution appraisal analyses. The original data set includes traveltimes from the experiments using A1 and B2 wells at BHRS. The receivers were in A1 and the transmitters in B2. The transmitter and receiver intervals were 0.2 m and 0.05 m, respectively. Since this data set included quite large amount of data, we produced a subset from these data to expedite inversion process. This subset was obtained by taking the experiments performed between 4 and 18 m and by taking every fourth traveltimes corresponding to those experiments. Thus, the data set we used has equal transmitter and receiver intervals of 0.2 m. The tomograms were obtained at the end of 15th iteration, and Broyden’s update was implemented after the third and for all subsequent iterations to improve computational efficiency further during the solutions. A constant velocity of 0.09 m/ns, which was calculated from the observed traveltimes, was used as starting model for the inversions and velocity updates were obtained by the LSQR approach. A depth interval between 4 m and 18 m were imaged. Table 3 lists the main parameters used for the inversions.

<table>
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<th>Iteration no</th>
<th>Traveltime residual (ns)</th>
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<th>λ = 4</th>
<th>λ = 6</th>
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<tr>
<td>1</td>
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</tr>
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</table>

The velocities (ranging 0.077–0.11 m/ns) in the tomograms (Fig. 14a, e) display a saturated zone characteristic indicated by low subsurface velocity contrast. The relatively high-velocity layer observed between 7 and 12 m is the main feature of the tomograms. As might be expected, the solution with λ value of 6 yielded a smoother velocity distribution with higher traveltime residual.
(0.26 ns) compared to the tomogram with $\lambda$ value of 4. The estimated uncertainties are less than 0.003 m/ns in each solution (Fig. 14b, f). Both high values of the uncertainties and low values of the model resolution are coincident and this is well observed at top and bottom parts of the imaged regions (Fig. 14b, c, f, and g). In order to increase the visibility of the ray coverage plot, the raypaths for every second
transmitter–receiver configuration were plotted (Fig. 14d, h). These plots display almost homogeneous ray coverages for either solution. Again, the unconstrained parts in the ray coverage plots are coincident with the parts of high values in the uncertainties and of low values in the model resolution images. Comparison of some of the observed and the calculated traveltimes is shown in Fig. 15. The calculated first-arrival times were obtained from the velocity distribution shown in Fig. 14a. The plots indicate a good match among the traveltimes. Fig. 16 shows a comparison between the tomogram in Fig. 14a and the one in Fig. 10d of Irving et al. (2007) (Courtesy of Society of Exploration Geophysicists). They used a ray-based inversion algorithm to obtain the velocity model shown in Fig. 16b. Although our solution displays some minor differences in velocity scale, it successfully reveals major features of the subsurface velocity distribution indicated by their solution. This comparison also reveals that the suggested inversion scheme was able to yield a velocity

Table 3

<table>
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<tr>
<th>Parameters</th>
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<th>Vertical</th>
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<tr>
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<td></td>
</tr>
<tr>
<td>Number of traveltimes</td>
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</tr>
<tr>
<td>Initial model velocity (m/ns)</td>
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Fig. 13. Velocity images for Model 3 resulting from the application of the CGLS scheme with a λ value of 4. (a) Noise-free data and (b) noisy data.

Fig. 14. Inverted model and inversion verification results for the field data (Courtesy of CGISS) at the end of 15th iteration with a λ value of 4 using the LSQR algorithm. The well A1 is on the left and well B2 on the right. (a) Velocity image (b) estimated model uncertainties (c) diagonal elements of the model resolution matrix and (d) ray coverage plot. (e)–(h) are similar to (a)–(d) but with a λ value of 6.
distribution as satisfactory as the one resulting from a standard ray-based tomography.

4. Conclusion

We showed an application of an eikonal-equation-based traveltime inversion of crosshole radar direct-arrival data. The proposed scheme yielded successful results with the tests using both synthetic and field data sets. Our approach was characterized by fast and stable convergence during the inversion processes. Using Broyden’s method for computing the Jacobian matrix noticeably improved the computational performance of the scheme. It is recommended to replace the explicit calculation of the Jacobian matrix by the numerical approximation after at least the second iteration. The solutions with the noisy synthetic data sets indicated that the tomograms were relatively insensitive to the errors in the data and in the model assumptions. We also observed that the LSQR algorithm produced better results than the CGLS did in the tests based on the complex subsurface models. This can be explained by that LSQR algorithm is more successful if the problem is ill-conditioned. Overall, the test studies indicated that the proposed scheme could be considered as an effective traveltime inversion technique for evaluation of crosshole radar data.

Acknowledgements

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