Tomography of the Darcy velocity from self-potential measurements

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[1] An algorithm is developed to interpret self-potential (SP) data in terms of distribution of Darcy velocity of the ground water. The model is based on the proportionality existing between the streaming current density and the Darcy velocity. Because the inverse problem of current density determination from SP data is underdetermined, we use Tikhonov regularization with a smoothness constraint based on the differential Laplacian operator and a prior model. The regularization parameter is determined by the L-shape method. The distribution of the Darcy velocity depends on the localization and number of non-polarizing electrodes and information relative to the distribution of the electrical resistivity of the ground. A priori hydraulic information can be introduced in the inverse problem. This approach is tested on two synthetic cases and on real SP data resulting from infiltration of water from a ditch.

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1. Introduction

[2] Geophysical methods such as ground-penetrating radar, DC electrical resistivity tomography, electromagnetic methods, induced polarization, seismic, and nuclear magnetic resonance imaging are sensitive to various hydraulic parameters of porous and fractured materials through the detection of changes in soil physical properties over time. The self-potential (SP) method is the only geophysical method that is directly sensitive to the flow of the ground water [e.g., *Revil et al.*, 2005]. Non-polarizing electrodes can therefore be considered as a non-intrusive flow sensor. *Jardani et al.* [2006] inverted recently SP data to reconstruct a boundary between two formations characterized by a net divergence of the streaming current density with the assumption that the vadose zone is more resistive than the aquifer. *Minsley et al.* [2007] proposed an algorithm to invert SP data in terms of the divergence of the streaming current density.

[3] As far as we know, the inversion of the streaming current density has never been attempted. The determination of this distribution would be very useful to determine the distribution of the Darcy velocity using the pore-scale electrokinetic model validated recently by *Bolève et al.* [2007a]. We propose below an algorithm to map ground water flow non-intrusively from SP data. This offers the possibility to monitor, in real time, ground water flow in the subsurface of the Earth. In this letter, we focus only on simple cases. We feel, however, applications to more complex systems will require the combination of SP inversion with additional geophysical methods and with in situ measurements of groundwater heads and/or temperature.

2. Forward Modeling

[4] Self-potential signals of electrokinetic nature are due to the drag of the excess of electrical charge contained in the pore water and resulting from the existence of the electrical diffuse layer at the pore water/mineral interface. In an isotropic but possibly heterogeneous medium, the total current density is given by [*Revil et al.*, 2007; *Bolève et al.*, 2007b]

$$\mathbf{j} = -\sigma(\theta)\nabla\varphi + \overline{Q}_V \frac{\theta_s}{\theta}\mathbf{u}, \tag{1}$$

where θ_s is the porosity, θ is the water content, $\mathbf{j}$ is the electrical current density (in A m^{-2}), $\mathbf{u}$ is the Darcy velocity (in m s^{-1}), φ is the SP (in V), σ is the electrical conductivity of the porous material (in S m^{-1}) [see *Revil et al.*, 1998], and $\overline{Q}_V$ is the excess charge (of the diffuse layer) of the pore water per unit pore volume (in C m^{-3}), which depends mainly on the permeability of the porous material (Figure 1). The continuity equation for the electrical charge is $\nabla \cdot \mathbf{j} = 0$. The Richards equation is $\mathbf{u} = -(k_r K_s)\nabla(H + z)$ [*Richards*, 1931], where k_r is the relative permeability, K_s the hydraulic conductivity at saturation, and $H = \delta p/\rho_f g$ is the change in hydraulic head at a given elevation (above or below the hydrostatic initial distribution H_0), δp is the change of pressure relative to the hydrostatic level, ρ_f is the pore fluid density (in kg m^{-3}), and g is the acceleration of the gravity (in m s^{-2}). At saturation, the streaming potential coupling coefficient is defined by $C = (\partial\varphi/\partial H)_{\mathbf{j}=0} = -\overline{Q}_V K/\sigma$. The measurement of C can be used to determine the values of $\overline{Q}_V$ [*Bolève et al.*, 2007b]. In Figure 1, we have reported the value of $\overline{Q}_V$ for different measurement of C. We observe that for a variety of rocks and ionic strengths of the pore water, $\overline{Q}_V$ depends mainly on the permeability of the porous rock.

[5] The hydraulic problem can be solved using the continuity equation,

$$[C_e + S_e S_S]\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{u} = 0, \tag{2}$$

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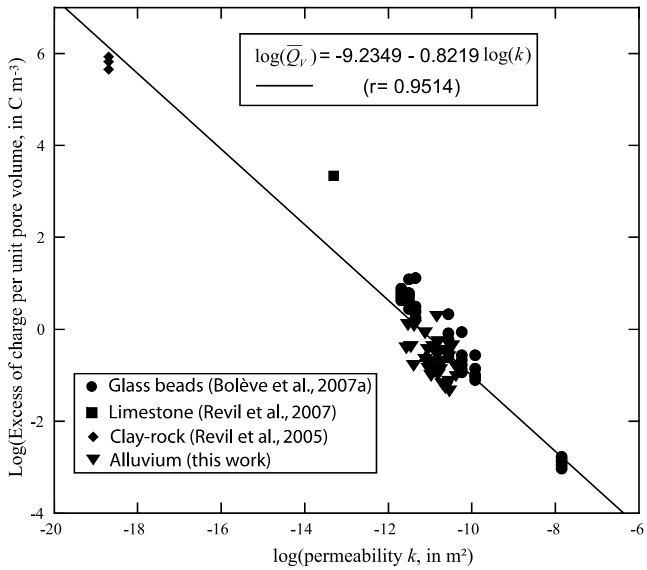

Figure 1. Dependence of the excess of charge per unit pore volume with the permeability at saturation of the porous materials. This data set includes various lithologies and salinities ($6 \leq \text{pH} \leq 8.5$). The alluvium materials are from the Boise State University test site. They are composed of a mixture of sands and gravels.

where H (m) is the pressure head, z is the altitude above a datum, C_e denotes the specific moisture capacity (in m^{-1}) defined by $C_e = \partial\theta/\partial H$ where θ is the water content (dimensionless), S_e is the effective saturation, which is related to the relative saturation of the water phase by $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$ where θ_r is the residual water content.

[6] With the van Genuchten parameterization, we consider the soil to be saturated when the fluid pressure reaches the atmospheric pressure ($H = 0$). The effective saturation, the specific moisture capacity, and the relative permeability are given by

$$S_e = \begin{cases} 1/\left[1 + |\alpha H|^n\right]^m, H < 0 \\ 1, H \geq 0 \end{cases} \tag{3}$$

$$C_e = \begin{cases} \frac{\alpha m}{1-m}(\phi - \theta_r)S_e^{\frac{1}{m}}\left(1 - S_e^{\frac{1}{m}}\right)^m, H < 0 \\ 0, H \geq 0 \end{cases} \tag{4}$$

$$k_r = \begin{cases} S_e^L\left[1 - \left(1 - S_e^{\frac{1}{m}}\right)^m\right]^2, H < 0 \\ 1, H \geq 0 \end{cases} \tag{5}$$

respectively, and α, n, $m = 1 - 1/n$, and L are dimensionless constants that characterize the porous material [*van Genuchten*, 1980]. *Bolève et al.* [2007b] used the commercial finite element software Comsol Multiphysics 3.3 to determine the SP distribution associated with ground water flow in saturated and unsaturated conditions under either steady-state or transient conditions (forward problem). The forward problem was validated with several data sets. In section 3, we invert the distribution of the current density $\mathbf{j}_S = \overline{Q}_V(\theta_s/\theta)\mathbf{u}$ from SP data.

3. Inverse Modeling

[7] The relationship between the electrical current density at point M and the measured SP signals at non-polarizing electrode P can be written as

$$\varphi(P) = \int_\Omega \mathbf{K}(P, M)\mathbf{j}_S(M)dV, \tag{6}$$

where $\mathbf{j}_S$ is the source current density (in both saturated and unsaturated conditions) described in section 2 and $\mathbf{K}(P, M)$ is the kernel connecting the SP data measured at a set of non-polarizing electrodes P (with respect to a reference electrode) and the source of current at point M in the conducting ground. The kernel $\mathbf{K}$ depends on the number of measurement stations at the ground surface, the number of discretized elements in which the source current density is going to be determined, and the resistivity distribution of the medium. The inversion of the SP data follows a two-step process. The first step is the inversion of the distribution of the current density $\mathbf{j}_S$. The second step is the determination of $\mathbf{u}$ using the distribution of $\mathbf{j}_S$ and assuming values for the excess charge density and the ratio (θ_s/θ) for unsaturated conditions.

[8] This SP inverse problem is a typical (vectorial) potential field problem and the solution of such problem is known to be ill-posed and non-unique. It is therefore important to add additional constraints to reduce the space of the solution. The criteria of data misfit and model objective function place different and competing, requirements on the models. These objective functions are balanced using Tikhonov regularization [*Tikhonov and Arsenin*, 1977] through the definition of a global objective function, ψ,

$$\psi = \|\mathbf{W}_d(\mathbf{Km} - \varphi_d)\|^2 + \lambda\|\mathbf{W}_m(\mathbf{m} - \mathbf{m}_0)\|^2, \tag{7}$$

where $\|\mathbf{Af}\|^2 = \mathbf{f}^t\mathbf{A}^t\mathbf{Af}$ (^{t} is transpose), λ is a regularization parameter under the constraint that $(0 < \lambda < \infty)$, $\mathbf{K} = (\mathbf{K}_{ij}^x, \mathbf{K}_{ij}^z)$ is the kernel Nx$2M$ matrix corresponding to the SP, which can be measured by each component of a source at coordinates $\mathbf{m} = (\mathbf{j}_i^x, \mathbf{j}_i^z)$ and where N is the number of SP stations while M is the number of discretized cells composing the ground, $2M$ represents the number of elementary current sources to consider (one horizontal component and one vertical component per cell for a 2D problem), φ_d is vector of N elements corresponding to the SP data measured at the ground surface or in boreholes, $W_d = \text{diag}\{1/\varepsilon_1,\ldots,1/\varepsilon_N\}$ is a square diagonal weighting NxN matrix (elements along the diagonal of this matrix are the reciprocals of the standard deviations ε_i of the data), $\mathbf{W}_m$

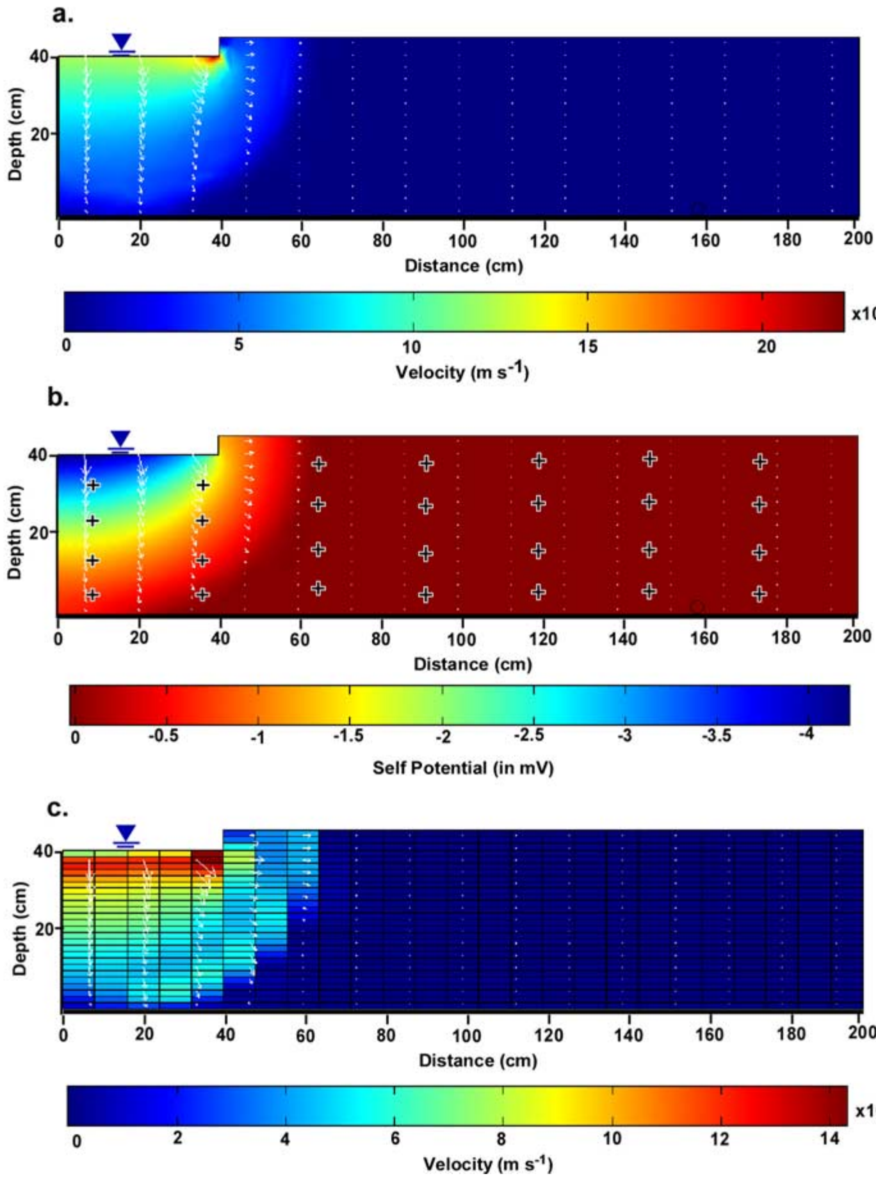

Figure 2. Synthetic case for the 2D-infiltration from a ditch. (a) The Darcy velocity is modeled from the Richard equation. (b) Distribution of the self-potential for the synthetic case. We assume that the «measurements» of the self-potentials are performed at the location with the symbols (+) are located. (c) Distribution of the reconstructed Darcy velocity (R^2 = 0.98).

is a $2(M-2) \times 2M$ weighting matrix (e.g., the flatness matrix or the differential Laplacian operator), $\mathbf{m}$ is the vector of $2M$ model parameters (source current density), and m_0 is a reference model (i.e., prior distribution of the source current density). $\mathbf{W}_m$ is given by *Zhdanov* [2002] and we consider that the probability distribution of the SP measurements is Gaussian [*Linde et al.*, 2007].

[9] The solution of the problem corresponding to the minimum of the cost function is [*Hansen*, 1998]:

$$\mathbf{m}^* = \left[\mathbf{K}^T\left(\mathbf{W}_d^T\mathbf{W}_d\right)\mathbf{K} + \lambda\left(\mathbf{W}_m^T\mathbf{W}_m\right)\right]^{-1} \cdot \left(\mathbf{K}^T\left(\mathbf{W}_d^T\mathbf{W}_d\right)\varphi_d + \lambda\left(\mathbf{W}_m^T\mathbf{W}_m\right)\mathbf{m}_0\right). \quad (8)$$

The solution depends on the value of the regularization parameter λ and the a prior model $\boldsymbol{m}_0$. To determine the value of λ, *Hansen* [1998] proposed plotting the norm of the regularized smoothing solutions versus the norm of the residuals of the data misfit function. This dependence often has an L-shaped form and the best regularization parameter lies on the corner of the L-shape curve. If we use a null distribution of prior information ($\boldsymbol{m}_0$ = 0), the previous model is similar to a damped weighted linear least squares or biased linear estimation problem. However, it is also possible to estimate the a prior model by simulating the flow of the ground water assuming an homogeneous subsoil, using the appropriate boundary conditions, and finally converting the seepage velocity in an a prior distribution of

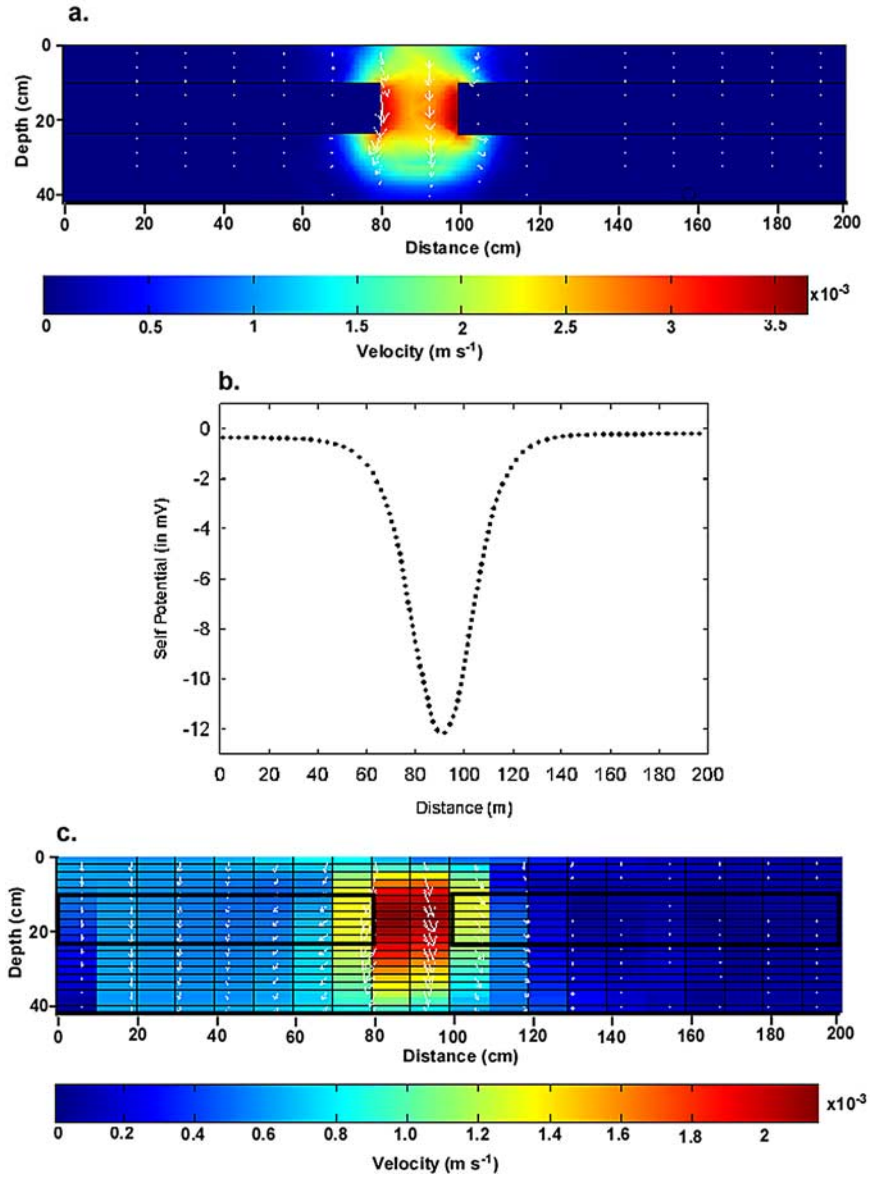

Figure 3. Synthetic case for a vertical 2D-infiltration problem. (a) True distribution of the Darcy velocity (in m s^{-1}). (b) Distribution of the resulting self-potentials, which is sampled at the top surface of the system every meter. (c) Distribution of the reconstructed Darcy velocity (R^2 = 0.95).

the current density using a constant value for $\overline{Q}_V$. We will use both approaches in the following examples.

4. Synthetic Cases

[10] The inverse model was set up in a Matlab routine. To test this routine, we simulated the case of the flow of the ground water from a ditch in a small thin tank with a length of 2 m and a height of 0.5 m. The fictitious tank is assumed to be filled with a porous material with constant properties (σ_{sat} = 0.012 S m^{-1}, ϕ = 0.33, $K_s = 8 \times 10^{-5}$ m s^{-1}, and $C_{sat} = -3$ mV m^{-1}). This yields a volumetric charge density $\overline{Q}_V$ = 0.48 C m^{-3} with η_f = 1.14 10^{-3} Pa s. The flux is imposed at the ditch (0.4 mm s^{-1}). The boundaries of the tank are both impermeable and insulating (**n.u** = 0 and **n.j** = 0). The Richards equation is solved with Comsol Multiphysics 3.3 [*Bolève et al.*, 2007b]. Figures 2a and 2b show the distribution of the Darcy velocity and the resulting SP field at t = 300 s. The material properties are α = 1.54, n = 7.6, θ_r = 0, and L = 0.5. We assume that the SP distribution is sampled at 28 non-polarizing electrodes (indicated by the crosses on Figure 2a). Then, this SP data are inverted to determine the distribution of the source current density using a null distribution of prior information (Figure 2c). Finally, we use the inverted distribution of the current density to determine the Darcy velocity. The distribution of the Darcy velocity is very similar to the modeled ground water flow pattern. The magnitude of the Darcy velocity is slightly smaller than the true Darcy velocity of the model.

[11] A second synthetic case is shown in Figure 3. This time, we model the vertical flow path due to an heterogeneity in the distribution of K_s. We have a uniform background medium ($K_s = 10^{-4}$ m s^{-1}, ϕ = 0.33, σ_{sat} =

0.012 S m^{-1}) and a less permeable layer (K_s = 10^{-7} m s^{-1}, ϕ = 0.40, σ_{sat} = 0.10 S m^{-1}) that contains a discontinuity. The flux is imposed from the top surface of the system (1 mm s^{-1}) and computations are performed in transient conditions with Comsol Multiphysics 3.3. The results are shown at t = 60 s (Figure 3). Then, the SP information determined at the ground surface is used to retrieve the position of the permeable pathway using a null distribution of prior information. We determine the distribution of the Darcy velocity (from its strength and the distribution of $\overline{Q}_V$, which is connected to the distribution of the permeability, Figure 1). Again the inverted values have a smoother distribution than the true Darcy velocity and a slightly smaller magnitude, but the final results compare well with the "true" model.

5. Application to an Infiltration Test

[12] We analyze now the infiltration experiment reported by *Suski et al.* [2006] and carried out at the test site of Roujan in the southern part of France. Eighteen piezometers were installed to a depth of 4 m on one side of a ditch, which is 0.8 m deep, 1.5 m wide, and 10 m long [*Suski et al.*, 2006]. The SP signals were monitored using a network of 41 PMS9000-Pb/PbCl$_2$ electrodes. Electrical resistivity tomography (ERT) along a section perpendicular to the ditch indicates that the resistivity of the soil was ~20 Ωm except for the first 50 cm where the resistivity was ~100 Ωm. The piezometers show that the water table was initially located at 2 m below the ground surface. During the experiment, 14 m^3 of fresh water were injected in the ditch. Laboratory experiments yields C = −5.8 ± 1.1 mV m^{-1}. The SP profile is can be found in the work by *Suski et al.* [2006]. Because of the symmetry of the problem with an axis of symmetry corresponding to the ditch, only one side of the ditch is modeled.

[13] To perform the inverse problem, the a prior model $\mathbf{m}_0$ is setup using the solution of the flow model at the time at which the self-potential measurements were obtained (170 minutes after the start of the infiltration). The result of the inversion is shown on Figure 4. Note that in total, the model uses 294 cells for its discretization but only the 64 cells close to the ditch are shown in Figure 4). A 2D numerical simulation was performed with Comsol Multiphysics 3.3 along a cross-section perpendicular to the ditch (Figure 5). We use the full formulation including capillary effects and heterogeneity in the distribution of the electrical resistivity [see *Bolève et al.*, 2007b]. Inside the ditch, we imposed a hydraulic head that varies over time according to the measured water level [*Suski et al.*, 2006]. The hydrogeological model and the values of $\overline{Q}_V$ used to perform the simulation is reported by *Bolève et al.* [2007b]. A snapshot of the SP and Darcy velocity distributions in the course of the infiltration is shown in Figure 5. The distribution of the Darcy velocities agrees reasonably well with the result inverted from the SP data (Figure 4).

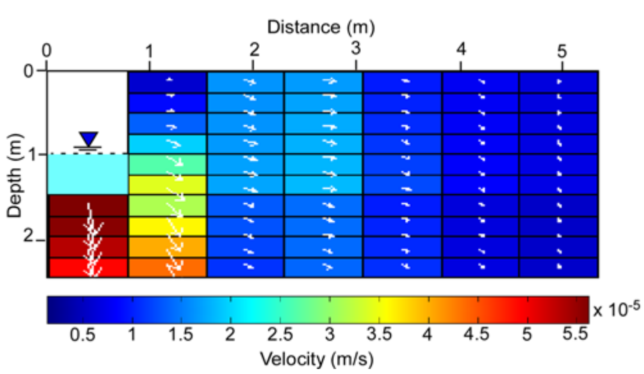

Figure 4. Reconstruction of the Darcy velocity from the self-potential data of *Suski et al.* [2006] (R^2 = 0.98) using an a prior model.

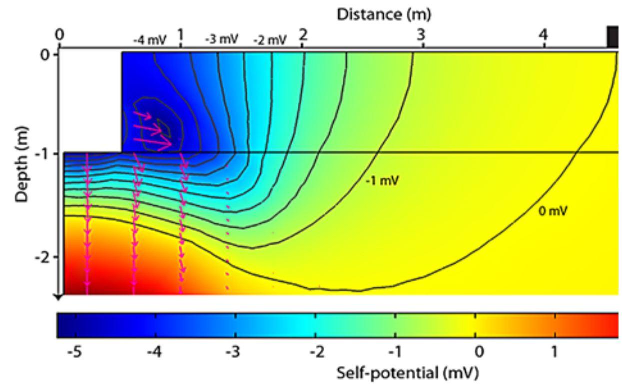

Figure 5. Simulated SP distribution and the Darcy velocity associated with the infiltration in the ditch of Roujan using all the hydraulic information available from in situ and petrophysical measurements [see *Bolève et al.*, 2007b].

6. Conclusion

[14] We used a finite element code (Comsol Multiphysics 3.3) within an inverse model based on Tikhonov regularization (in a Matlab routine) to consider the semi-coupled differential equations of fluid flow and electrical current density. The inversion of SP data with a constraint of smoothness allows the determination of the distribution of ground water flow velocity in the subsurface of the Earth for moderately heterogeneous media.

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