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Alternative linearization of water table kinematic condition for unconfined aquifer pumping test modeling and its implications for specific yield estimates

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SUMMARY

An alternative linearization of the nonlinear kinematic condition at the water table used in unconfined aquifer flow theory is proposed. It simulates the downward propagation of the water table, during water extraction via a pumping well, as a diffuse process controlled by a dimensionless linearization parameter, β_D. The limiting value of $\beta_D = 0$ corresponds to the solution of Neuman (1972) whereas that of $\beta_D \to \infty$ corresponds to the early-time Theis solution. Using the solution obtained with this linearization, data collected during a pumping test conducted at the Boise Hydrogeophysical Research Site in Idaho, US, were analyzed. The estimates of hydraulic conductivity and specific storage obtained in this study compare well with those obtained with the other models. The most significant improvement is in the estimated value of specific yield, with the proposed model yielding values that are close to those expected from known porosity values ($n \sim 0.25$) at the site. The results show that meaningful estimates of specific yield can be obtained by treating the water table as a diffuse moving material boundary, without recourse to more complex models for water table decline or inclusion of unsaturated zone flow.

1. Introduction

It has long been realized that if the water table is treated as a moving material boundary, the solutions of Neuman (1972), for a fully penetrating pumping well, and Neuman (1974), for a partially penetrating pumping well, tend to yield estimates of specific yield that are significantly smaller than those determined from more direct methods. For instance, Neuman (1975) obtained an average estimate of around 0.05 for the specific yield using drawdown data from a pumping test conducted in an unconfined aquifer consisting of medium-grained sands, gravel and a clayey matrix. From the composition of the aquifer material, one would expect the specific yield to be at least four times as large. In another field application of the model of Neuman (1972), Nwankwor et al. (1985) obtained estimates averaging 0.06 for the specific yield when they analyzed pumping test data obtained at the Borden research site in Canada. These values were significantly smaller than the average value of $S_y = 0.3$ that they obtained from the volume-balance and laboratory drainage methods. Nwankwor et al. (1992) proposed that the anomalous values of specific yield estimated with the model of Neuman (1972, 1974) were due to the model's inadequacy in describing the drainage process above the water table. They argued that the model, in effect, assumes instantaneous release of water from the unsaturated zone. They even conducted field experiments that demonstrated, contrary to the instantaneous drainage assumption, the gradual drainage of water from the unsaturated zone in response to water table decline during a pumping test conducted in the unconfined aquifer at the Borden research site.

In an effort to simulate the drainage process above the water table more correctly, Moench (1995, 1997) and Moench et al. (2001) proposed an empirical approach where the boundary condition at the water table used by Neuman (1972, 1974) was modified by incorporating the drainage mechanism espoused by Boulton (1954). In effect, whereas the flow model of Boulton (1954) is depth averaged, with term simulating water release from storage by water table displacement incorporated into the governing equation as a source term, Moench (1995, 1997) considers a two-dimensional axial-symmetric flow system that uses the source term of Boulton (1954) as the boundary condition at the water table. Though the approach, in its simplest form, involves introduction of an empirical parameter that has no clear physical meaning (Moench, 1995, 1997), it was shown to yield more realistic estimates of specific yield when drawdown data, obtained from several observation wells and piezometers during a pumping test conducted in a glacial outwash aquifer in Cape Cod, Massachusetts, were analyzed by Moench et al. (2001).

Recently, Tartakovsky and Neuman (2007) developed an analytical solution that generalizes that of Neuman (1972, 1974) by incorporating flow in the unsaturated zone above the water table. Flow in the unsaturated zone is described by a linearized form of

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the Richard's equation where the exponential model of Gardner (1959) is used for the constitutive moisture content–pressure head and unsaturated hydraulic conductivity–pressure head relations. According to this coupled flow model, the effect of flow in the unsaturated zone on unconfined aquifer flow is fully characterized by the nondimensional parameter $\kappa_D = \kappa b$, where κ is the constitutive exponent in the model of Gardner (1959), and b is the initial saturated thickness. In the limit as $\kappa_D \to \infty$, the model of Tartakovsky and Neuman (2007) reduces to that of Neuman (1972, 1974). Using data from Moench et al. (2001), Tartakovsky and Neuman (2007) were able to obtain an average specific yield of 0.18 and an average $\kappa_D = 8.1$. It should be noted, however, that even the model of Neuman (1972, 1974) yields similar values of specific yield, averaging 0.15 when applied to the data from the glacial outwash aquifer in Cape Cod reported in Moench et al. (2001). This seems to indicate that the model of Tartakovsky and Neuman (2007) leads to only modest improvements in estimated values of specific yield over that of Neuman (1972, 1974). In fact, an attempt by the author to estimate specific yield from drawdown data obtained in pumping tests conducted at the Boise Hydrogeophysical Research Site, indicates no improvement in estimates of specific yields obtained with the model of Tartakovsky and Neuman (2007) over those obtained with Neuman (1972, 1974) the two models yield virtually the same results.

An alternative hypothesis is proposed herein to explain the anomalous values of specific yield estimated from unconfined aquifer data using the model of Neuman (1972, 1974). It is argued here that the anomalous values of estimated specific yield are attributable to the manner in which the kinematic condition at the water table is linearized in the unconfined aquifer flow theory of Neuman (1972, 1974). The linearization of Neuman (1972), which is due to Dagan (1964), in effect completely neglects the nonlinear flux term at the water table. The water table remains a sharp interface between the vadose and saturated zones throughout a pumping test. In the linearization model proposed here, the nonlinear flux component at the water table is approximated by the divergence of flux at the water table, which constitutes treatment of the water table as a diffuse interface. The process of water release from storage due to water table displacement is controlled by a dimensionless linearization parameter, β_D. The solution obtained with this new linearization is only a slight modification of that of Neuman (1972). It is shown that for the limiting value of $\beta_D = 0$ the new solution corresponds to that of Neuman (1972), and for $\beta_D \to \infty$ it corresponds to the early-time Theis solution.

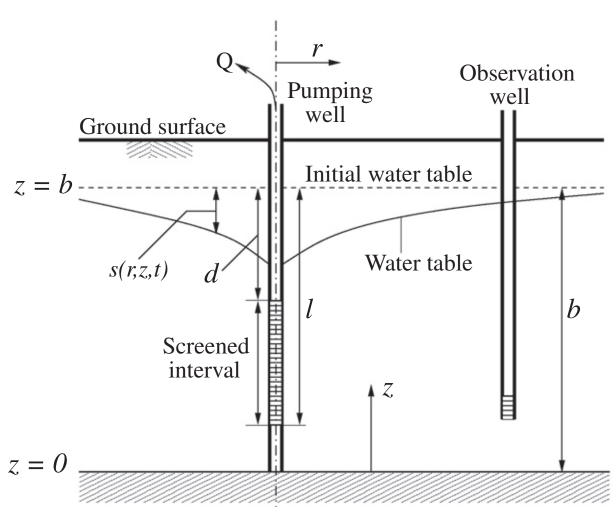

Fig. 1. Schematic of the unconfined aquifer conceptual model used in this work.

The solution leads to significant improvement in estimated values of specific yield over those obtained with the models of Neuman (1972, 1974) and (Moench, 1995, 1997). This is demonstrated by application of the solution to field data obtained in a pumping test conducted in the unconfined alluvial aquifer at the Boise Hydrogeophysical Research Site.

The main objective of this work is to demonstrate that improved estimates of specific yield can be obtained by a better linearization of the kinematic condition at the water table, without recourse to more complex models for water table decline or inclusion of unsaturated zone flow. Hence, in the following, with objective stated here in view, we start by outlining the theory of the proposed linearization, then derive the solution and conclude with application of the new solution to field data.

2. Theory

Flow to a continuous constant rate pumping well situated in a homogeneous anisotropic unconfined aquifer (see Fig. 1) is governed by the equation

$$S_s \frac{\partial h}{\partial t} = \frac{K_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) + K_z \frac{\partial^2 h}{\partial z^2} \tag{1}$$

where h is hydraulic head [L] (r,z) are radial and vertical coordinates [L], t is elapsed time since onset of pumping [T], K_r and K_z are aquifer radial and vertical hydraulic conductivities [L T^{-1}], and S_s is aquifer specific storage [L^{-1}]. Eq. (1) is typically solved subject to the initial condition:

$$h(r,z,0) = h_0, \tag{2}$$

and the farfield boundary condition

$$\lim_{r\to\infty} h(r,z,t) = h_0. \tag{3}$$

The boundary condition at the pumping well, taking into account partial penetration, is

$$\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right) = \begin{cases} \frac{Q}{2\pi(\ell-d)K_r} & \forall z \in [b-\ell, b-d] \\ 0 & \text{elsewhere}, \end{cases} \tag{4}$$

where Q is the pumping rate, and d and ℓ are the depths to the top and bottom, respectively, of the screened interval of the pumping well (see Fig. 1). A no-flow condition, given by

$$K_z \left.\frac{\partial h}{\partial z}\right|_{z=0} = 0, \tag{5}$$

is imposed at the base of the aquifer, where leakage from underlying water bearing formations is neglected.

The boundary condition at the water table needed to solve this flow problem in the absence of recharge, is given by the nonlinear kinematic condition (Bear, 1972)

$$S_y \frac{\partial h}{\partial t} = -K_z \frac{\partial h}{\partial z} + \mathbf{K}\nabla h \cdot \nabla h, \tag{6}$$

where S_y is the specific yield and $\mathbf{K}$ is the aquifer conductivity tensor. The right-hand-side of Eq. (6) reflects the fact that the flux vector at the water table is not entirely vertical since the cone of depression, bounded by the water table, propagates both downward and radially. It also indicates that the flux at the water table comprises a linear (first term) and a nonlinear (second term) flux component. When Eq. (6) is linearized according to the method of Dagan (1964), as was done in Neuman (1972), it becomes

$$S_y \left.\frac{\partial h}{\partial t}\right|_{z=b} = -K_z \left.\frac{\partial h}{\partial z}\right|_{z=b}. \tag{7}$$

where b is the initial saturated thickness of the aquifer (see Fig. 1). The linearization given by Eq. (7), in effect, implies flow conditions such that the nonlinear flux term is negligibly small, i.e. $\nabla h \cdot \nabla h \approx 0$ (Bear, 1972). It is important to remember that the water table position is fixed at its original position and it remains a sharp interface between the vadose and saturated zones. When the solution based on this model of water release at the water table, it has been found, starting with Neuman (1975), that it significantly underestimates the specific yield, S_y.

As a first approximation of the nonlinear flux term, to account for flow conditions where this term may not be completely negligible, an alternative linearization of Eq. (6) is proposed in this work. Thus, instead of neglecting the nonlinear term in Eq. (6) entirely, one can rewrite it as

$$\nabla h \cdot \nabla h = -h\nabla^2 h + \frac{1}{2}\nabla^2 h^2. \tag{8}$$

Ignoring the second term yields $\nabla h \cdot \nabla h \approx -h\ \nabla^2 h$, which we linearize as $\nabla h \cdot \nabla h \approx -\beta \nabla^2 h$ where β is a linearization parameter. Hence, the linearized kinematic condition at the water table, neglecting the horizontal flux components, becomes

$$S_y \left.\frac{\partial h}{\partial t}\right|_{z=b} = -K_z \left.\left(\frac{\partial h}{\partial z} + \beta \frac{\partial^2 h}{\partial z^2}\right)\right|_{z=b}. \tag{9}$$

The second term on the right-hand-side of Eq. (9) has the form of a diffusion term and partly accounts for the effects of the nonlinear fluxes at the water table under flow conditions where they may not be neglected entirely. With this model, the water table may be thought of as being a diffuse interface, where water release is controlled both by specific yield and the linearization coefficient, β. The effect of the parameter on the solution and model predicted response is considered in the following section.

To solve the flow problem as described above we first rewrite it in terms of drawdown, $s = h(r,z,0) - h(r,z,t)$, and non-dimensionalize it by defining non-dimensional variables $s_D = s/H_c, r_D = r/b, z_D = z/b$ and $t_D = \alpha_r t/b^2$, where $H_c = Q/(4\pi b\ K_r)$ and $\alpha_r = K_r/S_s$. The solution is then obtained using Laplace (in t_D) and Hankel (in r_D) transforms for flow toward a pumping well in an unconfined aquifer of infinite radial extent, with no leakage.

2.1. Fully penetrating pumping well solution

It can be shown that the solution for the case of a fully penetrating pumping well is

$$\bar{s}^*_D(a, z_D, p) = \bar{u}^*_{D_f}(a,p)\left(1 - \frac{\cosh(\eta z_D)}{\Delta}\right) \tag{10}$$

where $\bar{s}^*_D$ is the Laplace and Hankel transform of dimensionless drawdown, s_D, a and p are Hankel and Laplace transform parameters, respectively,

$$\bar{u}^*_{D_f}(a,p) = \frac{2}{p(p+a^2)} \tag{11}$$

$$\Delta = (1 + \beta_D \eta \xi)\cosh(\eta) + \xi \sinh(\eta) \tag{12}$$

$\xi = \eta\alpha_D/p$, $\alpha_D = \kappa/\sigma$, $\kappa = K_z/K_r$, $\sigma = S_y/(bS_s)$, $\eta^2 = (p + a^2)/\kappa$ and $\beta_D = \beta/b$, the non-dimensional form of the linearization parameter. It is clear from Eq. (9) that one recovers the solution of Neuman (1972) for $\beta_D = 0$. In fact the solution in Eq. (10) looks exactly as the solution of Neuman (1972) except for the extra term $\beta_D\eta\xi$ in Eq. (12).

2.2. Partially penetrating pumping well solution

For the case of a partially penetrating well, it can be shown that the solution is (Malama et al., 2008)

$$\bar{s}^*_D(a, z_D, p) = \bar{u}^*_{D_p}(a, z_D, p) - \frac{\bar{u}^*_{D_p,0}}{\Delta}\cosh(\eta z_D) \tag{13}$$

where $\bar{u}^*_{D_p,0} = \bar{u}^*_{D_p}|_{z_D=1}$,

$$\bar{u}^*_{D_p} = \frac{\bar{u}^*_{D_f}}{l_D - d_D}\begin{cases} g_1(\eta, z_D) - g_2(\eta, z_D) & \forall z_D \in (1-d_D, 1] \\ 1 - g_2(\eta, z_D) & \forall z_D \in [1-\ell_D, 1-d_D], \\ f_3(\eta)\cosh(\eta z_D) & \forall z_D \in [0, 1-\ell_D) \end{cases} \tag{14}$$

$$g_1(\eta, z_D) = \cosh[\eta(1 - d_D - z_D)], \tag{15}$$

$$g_2(\eta, z_D) = \frac{f_1(\eta)\cosh(\eta z_D) + f_2(\eta)\cosh[\eta(1-z_D)]}{\sinh(\eta)}, \tag{16}$$

$$f_1(\eta) = \sinh(\eta d_D), \tag{17}$$

$$f_2(\eta) = \sinh[\eta(1-\ell_D)], \tag{18}$$

$$f_3(\eta) = e^{-\eta(1-\ell_D)} - \frac{f_1(\eta) + e^{-\eta} f_2(\eta)}{\sinh(\eta)}, \tag{19}$$

This solution corresponds to that of Neuman (1974) for $\beta_D = 0$ and to that of Hantush (1964) for $\beta_D \to \infty$. Additionally, for $d_D = 0$ and $\ell_D = 1$, one recovers Eq. (10), the solution for a fully penetrating aquifer.

2.3. Effect of β_D on predicted system response

From the theory presented above the linearization parameter $\beta_D \in [0,1]$. However, the model developed above can admit values of β_D outside this range. Hence, this parameter is hereafter treated as an empirical parameter such that $\beta_D \in [0,\infty)$. Its effect on drawdown is shown in Fig. 2a, where dimensionless drawdown, s_D, is plotted against dimensionless time, t_D/r_D^2, for different values of β_D. Fig. 2b, which shows model results obtained with the model of Moench (1997) for one fitting parameter α, is included for comparison. It can be seen from these results that, as expected from Eq. (9), one recovers the solution of Neuman (1972) for $\beta_D = 0$. Additionally, the results show that, in the limit as $\beta_D \to \infty$, the effect of the water table vanishes and the solution approaches the confined aquifer solution (Theis, 1935).

For non-zero and finite values, increasing the value of β_D has the effect of increasing the slope of the log–log drawdown curves as well as the drawdown in the intermediate-time range between the early-(labeled $E_1(x)$ in Fig. 2) and late-time (labeled $E_1(x_y)$ in Fig. 2) Theis curves. This is in clear contrast to the effect of the parameter α of the model of Moench (1997), which, as can be seen in Fig. 2b, tends to only alter the drawdowns but not the slope of the drawdown curve in the intermediate-time range. Hence, increasing the values of the α-parameter only has the effect of shifting the drawdown curves upward in the intermediate-time range, but has no effect on the position of the late-time Theis curve; all the curves for the different α values fall on the same late-time Theis curve. To shift the late-time curve, one has to change the specific yield, and in most field applications, this implies using anomalously low values of specific yield.

On the other hand, increasing the values of the parameter β_D has the effect of shifting the late-time Theis curve to the left. Note that the curves in Fig. 2a for the different values of β_D do not converge on the same Theis curve at late-time, even when S_y is fixed. The sensitivity of the apparent late-time Theis curve to the parameter β_D for fixed values of S_y, as well as its effect on the slope of the drawdwon curve during the intermediate-time range explain why the solution developed here yields better estimates of specific yield than the models of Neuman (1972, 1974) and Moench (1997) for the field data analyzed in Section 3 below. Fig. 3 shows the solution developed herein for the case of a partially penetrating pumping well plotted for different values of β_D, and is included here for completeness. The effects of β_D are essentially the same for a partially or fully penetrating pumping well.

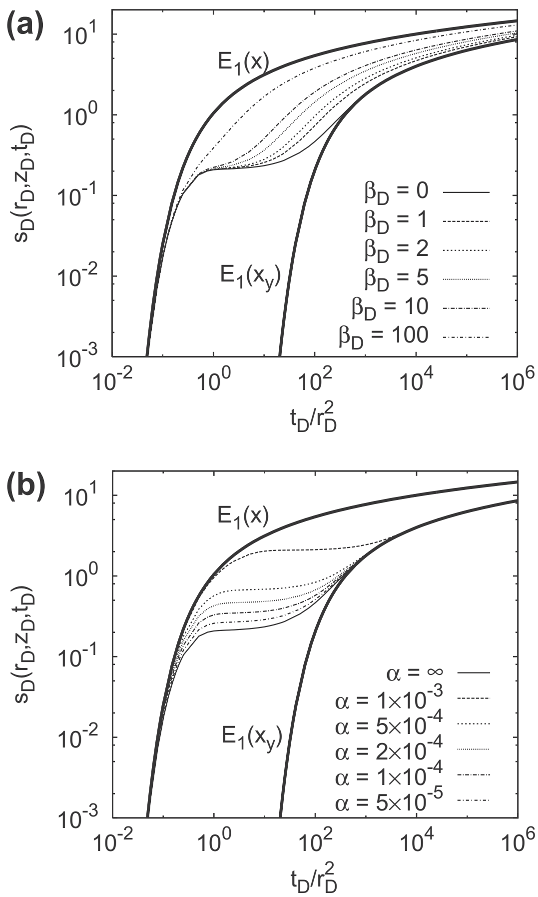

Fig. 2. Plot of dimensionless drawdown against t_D/r_D^2 for different values of the (a) dimensionless parameter β_D of the model proposed herein, and (b) the parameter α of the model of Moench (1997). Note that the curve for $\beta_D = 0$ in (a) and $\alpha = \infty$ in (b) corresponds to solution of Neuman (1972).

3. Model application to field data

3.1. Site and aquifer test description

We consider pumping test data obtained in the summer of 2008 at the Boise Hydrogeophysical Research Site (BHRS) in Boise, Idaho (US). The aquifer at the BHRS is unconfined and is bounded to the southwest by the Boise River, and below by a clay unit, which is continuous at the site. The aquifer, with a vertical extent (saturated thickness) of about 16 m, consists of unconsolidated cobble and sand fluvial deposits (Barrash and Clemo, 2002; Barrash et al., 2006). The depth to the water table before onset of pumping averaged 2 m. There were no observed precipitation events during the few days before the pumping test was conducted.

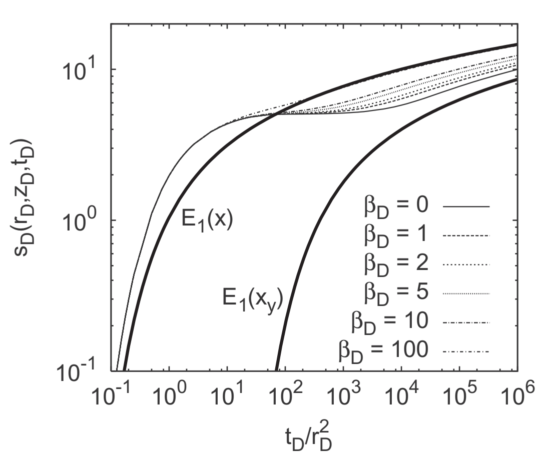

Fig. 3. Plot of dimensionless drawdown against t_D/r_D^2 for different values of the dimensionless parameter β_D for the proposed model using a partially penetrating pumping well.

Previous efforts to estimate aquifer parameters at the site using the method of Moench (1997), as implemented in the code WTAQ (Barlow and Moench, 1999), have yielded anomalously low values of specific yield (Barrash et al., 2006), with no significant improvement over those obtained with the model of Neuman (1972, 1974). Using the method of Tartakovsky and Neuman (2007) also yields results that are essentially the same as those obtained with the model of Neuman (1972). Hence, here we analyze data from a series of tests performed at the BHRS using the solution outlined above. In these tests, the aquifer was pumped continuously for about two hours at a constant rate of 2.84×10^{-3} m^{-3}/*s* (45 gpm) through a fully penetrating well. Head measurements were recorded in fully penetrating observation wells. All the wells at the site have the same radius 5.08×10^{-2} m (2 in). In the analysis, wellbore storage effects in observation wells were accounted for in the manner of Black and Kipp (1977) for all the models used to estimate aquifer parameters. The radial positions of the observation wells from the pumping well are given in Table 1.

3.2. Parameter estimation

Hydraulic parameter values, namely K_r, κ, S_s and S_y, were estimated with the parameter estimation software PEST Doherty (2002). The parameter κ here represents the anisotropy ratio, i.e. $\kappa = K_z/K_r$. For the model of Moench (1997), the empirical parameter α was also estimated. For the model proposed here, it should be noted that the linearization was obtained by setting the term $h\nabla^2 h \approx \beta\nabla^2 h$, which implies that $\beta \approx h$. Hence, for small drawdowns, $\beta \approx h(r, t = 0)$ and $\beta_D \approx 1.0$. Therefore, in the parameter estimation exercise, we estimated the parameter β_D using data from well B2 but set it to 1.0 for data from the other wells. Tables 1–3

Table 1
Hydraulic parameter values estimated with the model of Neuman (1972).

Well	r (m)	K_r (10^{-4} m/s)	S_s (10^{-4} m^{-1})	κ	S_y
B2	3.51	4.64	2.22	1.31	0.052
B4	2.62	4.61	3.48	2.02	0.090
B5	3.65	4.64	2.16	1.25	0.062
B6	2.69	4.57	2.52	1.08	0.080

Table 2
Hydraulic parameter values estimated with the model of Moench (1997).

Well	K_r (10^{-4} m/s)	S_s (10^{-4} m^{-1})	κ	S_y	α
B2	4.64	2.22	1.32	0.052	5.36×10^{-1}
B4	4.63	3.51	1.90	0.083	2.30×10^{2}
B5	4.64	2.16	1.25	0.062	2.84×10^{0}
B6	4.57	2.52	1.08	0.080	2.61×10^{1}

Table 3
Hydraulic parameter values estimated with the model proposed herein.

Well	K_r (10^{-4} m/s)	S_s (10^{-4} m^{-1})	κ	S_y	β_D
B2	4.26	2.11	1.93	0.197	1.47
B4	4.26	3.34	2.89	0.266	1.00
B5	4.26	2.08	1.75	0.185	1.00
B6	4.09	2.52	1.72	0.283	1.00

shows the parameter values estimated using the model of Neuman (1972), Moench (1997) and that developed herein, respectively. The model fits to the data are shown in Fig. 4 for data collected in observation wells B4 and B6. The model fits to data from wells B2 and B5 are comparable to those shown in the figure. To provide a measure of the goodness-of-fit, the R^2 values are included in the plots.

3.3. Discussion of parameter estimation results

All the models fit the pumping test drawdown data well with R^2-values of 1.0 for all the data. The estimates of hydraulic conductivity and specific storage obtained with all the models are comparable. The estimated values of the anisotropy ratio, κ, given in Tables 1–3 are larger than 1.0, for the most part. The aquifer material at the research site comprises unconsolidated fluvial deposits with significant root growth from the native cottonwood trees. This may explain why the values of κ estimated in this work are generally larger than 1.0. It may also be possible to attribute these anisotropy ratios to model limitations associated with the assumption zero initial head gradient and negligible vadose zone effects. Nevertheless, even for cases where κ was set equal to 1.0, the estimated values of the other parameters discussed here did not show significant change from those presented in Tables 1–3; setting $\kappa \equiv 1.0$ only had the effect of increasing the value of the isotropic hydraulic conductivity of the aquifer.

As one would expect, the estimates of specific yield (Table 1) obtained with the model of Neuman (1972) are significantly lower than the known porosity values at research site from neutron logging of the wells reported in Barrash and Clemo (2002). The porosity values are known to be in the range 0.12–0.40 (Barrash and Clemo, 2002; Barrash and Reboulet, 2004) whereas the specific yield values

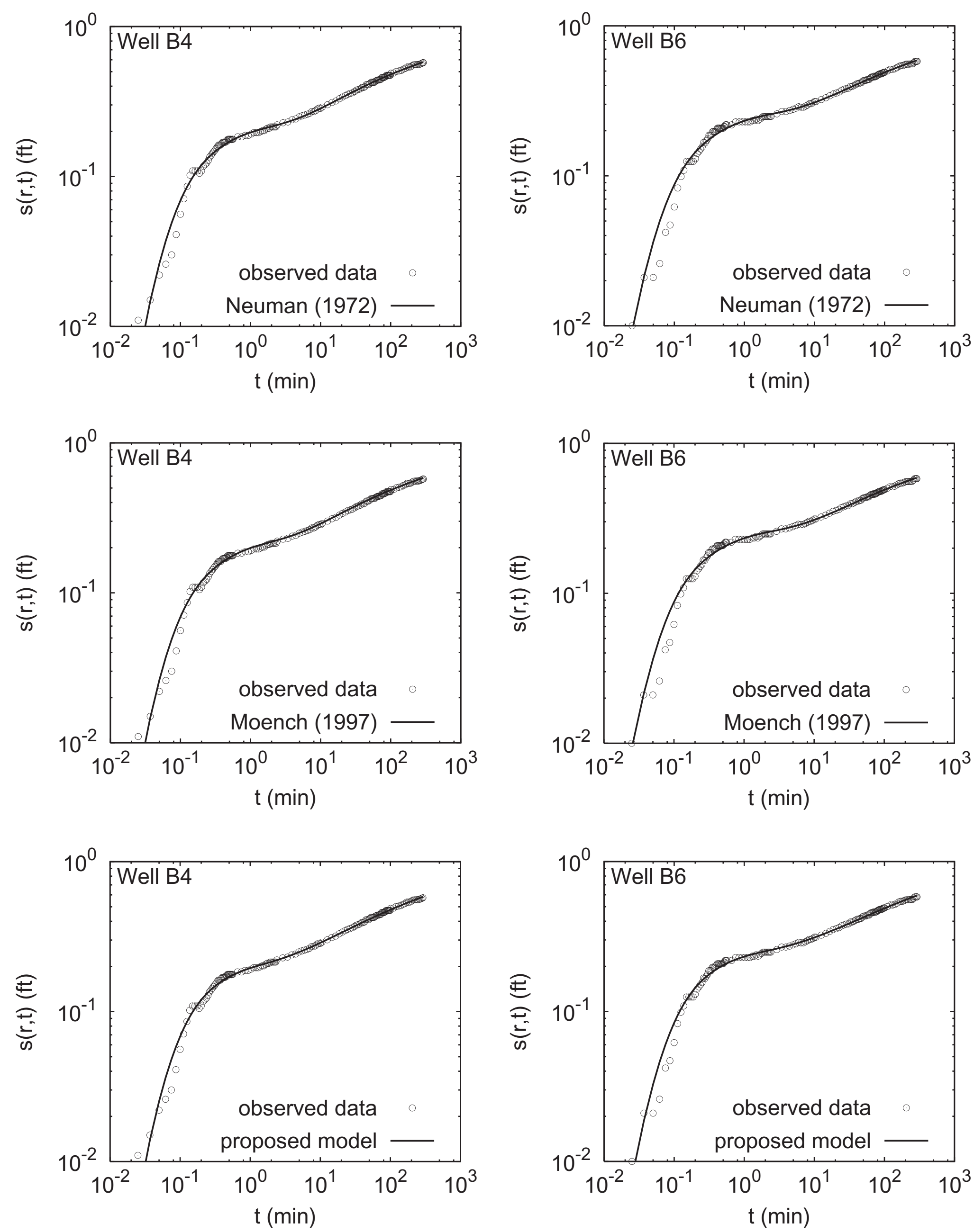

Fig. 4. Fits of the models of Neuman (1972), Moench (1997) and that proposed in this work to measured drawdown obtained during pumping tests at the Boise Hydrogeophysical Research Site.

estimated here with the model of Neuman (1972) are all less than 0.10. That such anomalously low values of specific yield are to be expected with the model of Neuman (1972), is well documented in the hydrogeology literature (Neuman, 1975; Barrash et al., 2006), and they cannot be explained by high specific moisture retentions of the sediments, as the sediments at the site are cobbles and sands with minimal fines (Barrash and Reboulet, 2004).

When the model of Moench (1997) was used to estimate aquifer parameters, it was found to yield values that similar to those obtained with the model of Neuman (1972) for the data analyzed in this work, with no improvement in estimates of specific yield (see Table 2). This was also found to be the case by Barrash et al. (2006) for other pumping tests conducted at the BHRS. Here, we have only allowed for one fitting parameter in the model of Moench (1997) to simulate water release due to water table decline. One may possibly improve estimates of specific yield with this model by including additional fitting parameters permitted by the model. However, it is difficult to ascribe physical meaning to such parameters. Additionally, analysis of the field data with the model of Tartakovsky and Neuman (2007) as implemented in AQTESOLV, did not yield any appreciable improvement in the estimated values of specific yield.

The model developed herein yielded significantly improved estimates of specific yield over the models of Neuman (1972) and Moench (1997). This can be seen by comparing the specific yield values reported in Tables 1 and 2, to those in Table 3. The values in Table 3 are consistently larger than those obtained with the models of Neuman (1972) and Moench (1997), and averaged $S_y = 0.23$. These larger values are well within the range of values expected from porosity measurements (neutron logging) for the alluvial deposits characteristic of the BHRS aquifer as reported in Barrash and Clemo (2002). The values of the linearization parameter, $\beta \approx 1.0$, used in the analysis are also consistent with what would be expected from the theory outlined above.

The improved values of S_y estimated with the model developed herein are due to the fact that the parameter β_D has the effect of shifting the late-time Theis curve ($E_1(x)$) to the left when its values are increased. This sensitivity of the late-time Theis curve to the parameter β_D allows for the model to be fitted to drawdown data with relatively large values of specific yield. It implies that one can shrink the separation between the early- and late-time Theis curves, whilst maintaining a relatively large value of specific yield. The parameter β_D does not only affect the values of drawdown in the intermediate-time range, but also the slope of the drawdown-time curve in that range. As discussed above, the α-parameter of the model of Moench seems to only affect the drawdown values in the intermediate-time range.

4. Conclusion

A significant limitation of the model of Neuman (1972, 1974) has long been recognized to be the underestimation of specific yield (Neuman, 1975). The works of Moench (1997) and Tartakovsky and Neuman (2007) were developed to address this shortcoming by accounting for slow drainage and flow in the unsaturated zone associated with water table displacement during unconfined aquifer pumping tests. These models have been shown to yield significant improvement in estimated values of specific yield for field data collected at Cape Cod (Moench et al., 2001; Tartakovsky and Neuman, 2007). However, attempts to apply these models to data collected at the Boise Hydrogeophysical Research Site, have failed to yield improvements in estimated values of specific yield (Barrash et al., 2006).

Hence, it has been argued here that the anomalously low specific yield value estimated with the model of Neuman (1972, 1974) maybe attributed to the first-order linearization of the kinematic condition at the water table. Whereas the linearization of the kinematic condition at the water table given in Eq. (7) and used by Neuman (1972), has been said to describe instantaneous water table decline during pumping, with the water table propagating downwards as a sharp interface between the saturated and unsaturated zones (Moench, 1997), the linearization proposed in Eq. (9) describes the downward propagation of the water table as a diffuse process. This diffuse downward propagation of the water table is controlled by the dimensionless linearization parameter, β_D, whose values, as suggested by the theory, should be close to 1.0. This parameter may, however, be treated empirically and estimated directly from the data.

The effect of the new parameter, β_D, is most evident in the intermediate-time range, where it alters, not only the values of drawdown, but also the slope of the drawdown curve. This has the effect of shifting the apparent late-time Theis to the left as the values of β_D increase. This leads to improved estimates of specific yield when the solution applied to the inversion of drawdown data. For the particular case of field data collected during pumping tests conducted at the Boise Hydrogeophysical Research Site (BHRS), it is clear that, whereas the model of Moench (1997) yields estimates of specific yield that are not significantly different from those obtained with the model of Neuman (1972), the model developed here yields values that are similar to those predicted on the basis of porosity estimates for the alluvial deposits at the site. It should be noted, however, that the anisotropy ratios estimated with the new model are on average larger than those estimated with the other two models. This should not be taken to be the reason for the better values of specific yield since fixing $\kappa = 1$ did not significantly alter the differences in the estimated values of specific yield. The results obtained herein show that meaningful estimates of specific yield can be obtained by treating the water table as a diffuse moving material boundary, without resorting to more complex models for water table decline or inclusion of unsaturated zone flow.

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