Chapter 8

Near-Surface Seismology: Surface-Based Methods

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Introduction

Seismic methods are geophysical techniques that involve the generation and recording of seismic waves for the purpose of mapping the subsurface. Each method is based on the propagation of waves from an artificial source to a set of receivers, followed by an analysis of the recorded wavefield in terms of subsurface properties. Although seismic methods are conceptually not limited to any particular macroscopic scale, the emphasis here is on source-receiver separations that range from a few meters to a few hundred meters, and on depths of investigation that fall approximately within the same range. These linear dimensions define the domain of near-surface seismology, and focus attention on a portion of the uppermost crust that is of great importance to other geologic disciplines, especially geotechnical engineering and hydrogeology. Site characterization and the delineation of aquifers constitute the primary practical applications of near-surface seismology, thereby explaining the many references to engineering and groundwater methods in the applied geophysical literature.

For much of its early history, near-surface seismology was dominated by analog seismographs and the seismic refraction method, with depth-to-bedrock probably being the most common objective. However, the introduction of affordable multichannel digital seismographs in the 1980s, along with the remarkable decrease in cost of powerful desktop computers, has led near-surface seismologists to embrace a broader variety of methods, including the seismic reflection method, the surface-wave method, and various borehole methods. The application of these methods to near-surface problems has necessarily been accompanied by an increased emphasis on numerical modeling of seismic wave propagation and the application of inverse theory. As a result, near-surface seismology is capable of routinely providing more complete information on shallow structure and stratigraphy than ever before. It is likely that the most important advances in the coming years will involve wider application of 3D imaging, progress in relating seismic measurements to geotechnical or hydrologic properties, and enhancements in data acquisition that lead to lower costs.

This chapter provides a survey of those near-surface seismic methods that are based on the deployment of sources and receivers at the surface on land. It builds on the review and tutorial articles by Steeples and Miller (1990) and by Lankston (1990), which appeared in the original SEG publication on geotechnical and environmental geophysics (Ward, 1990). The tradition established by those authors is continued here by pulling together essential concepts and presenting them at a uniform level with consistent notation and terminology. Although many details must remain unstated because the subject area is extensive, a lengthy list of cited references is provided to alleviate this difficulty.

The next section discusses the essential aspects of sources, receivers, and digital seismographs that are common to most field experiments. Separate sections are then devoted to each of the surface-based seismic methods that currently find significant use in a near-surface context on land — the refraction method, the reflection method, and the surface-wave method. A supplementary reference list is included to guide the reader to many of the peer-reviewed journal articles on near-surface seismology that have appeared in the interval 1990–2004 inclusive.

Seismic Sources and Field Instrumentation

In general, a seismic method is most effective when the seismic wave of interest is recorded with a broad spectral bandwidth and large signal-to-noise ratio. Although these desirable signal characteristics are limited by the physics of wave propagation in the earth and the ubiquity of noise, much has been done in the design of seismic sources and field instrumentation to enhance the quality of seismic data. This section provides an overview of commonly available sources and instrumentation for a general surface-based seismic field experiment on land. Important general references for this section include the treatises on exploration seismic instrumentation by Anstey (1970),

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Seismic field experiments

Surface-based seismic experiments conducted on land require a source to generate seismic waves, one or more receivers, and a digital recording system. The source is deployed at a specific location on the earth’s surface or in a shallow borehole. It generates a transient seismic wavefield that excites receivers deployed at points on the surface that are (usually) in the far-field. The geometric pattern of the source and the receivers on the surface is called the spread. The spread may be arranged along a straight or curved line or it may be distributed areally over any portion of the surface. The horizontal distance between a source and receiver is called the offset for that source-receiver pair. An inline offset spread is arranged along a straight line with the source located at one end but separated from the nearest receiver by nonzero offset; the inline offset spread is said to be reversed if sources are located at both ends. A split spread is arranged along a straight line with the source located at the center of the receivers. Many other types of spreads are possible; Figure 1 schematically illustrates some examples.

A receiver is an electromechanical transducer (or set of interconnected transducers) that is fixed to the earth and converts ground motion to an analog electrical voltage called the seismic signal. For each source effort, the seismic signal from each receiver is digitally recorded by a recording system called a seismograph. The digitally recorded seismic signal from a given receiver for a given source effort is called the recorded signal for that source effort and receiver; the term trace is commonly used to refer to the recorded signal or its analog equivalent. The collection of all traces from a given source effort is called a field record and is an example of a common-source gather or common-shot gather. The source effort may be repeated multiple times at each of multiple source locations for multiple deployments of the receivers until sufficient data are collected for analysis. Although there is considerable variation, the parameters for most near-surface seismic experiments on land will fall within the range of 1–1000 field records, with each record consisting of 1–120 traces, and each trace consisting of 1000–10,000 digital samples.

Sources

An ideal seismic source is a safe, cost-efficient, and repeatable generator of a seismic wavefield of broad spectral bandwidth and adequate signal strength at each receiver. The development of a source approaching these characteristics is a difficult problem that has been addressed by many seismologists and engineers. Of the numerous sources now available, most can be classified as one of the following: a single impulsive source, a swept frequency source, or a multiple impulsive source.

A significant number of source comparison tests have been published to assess source characteristics in various geologic settings (Miller et al., 1986; Miller et al., 1992; Miller et al., 1994; Buhmann and Holiger, 1998; Doll et al., 1998; Herbst et al., 1998; van der Veen et al., 2000). Perhaps the most important findings of these source tests and the many years of cumulative experience of near-surface seismologists are that the relative performance of sources is site dependent, and that the best performance of a given source is typically observed at sites characterized by fine-grained water-saturated sediments.

A single impulsive source imparts a force of short duration across a small area at the earth’s surface or within a small volume located inside the earth. These sources include chemical explosives, weight drops (both free falling and accelerated), projectiles from firearms, sudden releases of compressed air or water, and electrical discharges. The most commonly used single impulsive source is the sledgehammer which offers the advantages of low cost, portability, ease of use, and relative safety. Keiswetter and Steeples (1995) studied the source parameters for a sledgehammer. They found that the area of the striking plate and the hammer mass are more important in affecting source performance than hammer velocity or plate mass, and that the effects depend on the site. Projectile sources are also common and include vertically oriented rifles and shotguns fired in boreholes or at the surface (e.g., Steeples, 1984; Pullan and MacAulay, 1987; Parker et al., 1993). Baker et al. (2000) found in a test of sledgehammer and projectile sources (4.5-kg sledgehammer, 30.06 rifle, and .22-caliber rifle) at two sites that the least energetic source (.22-caliber rifle) generated more desirable high-frequency characteristics (above 250 Hz) than did the more energetic sources.

At the high-energy end of the available single impulsive sources in near-surface seismology, Liberty (1998) compared a compressed-air source (75-in³ land air gun) to...
where \( t \) of the impulse response can be isolated as input, and further suppose a portion \( s \) of receivers, or the recording system. This practice is known as \textit{vertical stacking} or \textit{field summing}, and is based

\[ s(t) = p(t) * r(t) + n(t). \]  

(1)

Source and propagation effects and the effect of the recording system are not shown in equation (1) for simplicity; see Yilmaz (2001, 219–222) for a more complete convolutional model. Crosscorrelation of \( s(t) \) with \( p(t) \) gives a modified signal \( s'(t) \):

\[ s'(t) = p(t) \otimes s(t) = [p(t) \otimes p(t)] * r(t) + p(t) \otimes n(t). \]  

(2)

where the operator \( \otimes \) indicates crosscorrelation (or autocorrelation for like signals); see Appendix A. If the bandwidth of the noise does not significantly overlap the bandwidth of the pilot signal, then by the crosscorrelation theorem (see Appendix A), the noise crosscorrelation is relatively small, and equation (2) becomes

\[ s'(t) = k(t) * r(t) + n'(t). \]  

(3)

where \( k(t) = p(t) \otimes p(t) \) is the autocorrelated pilot signal (sometimes called the \textit{Klauder wavelet}), and \( n'(t) = p(t) \otimes n(t) \) is the remainder from the noise crosscorrelation. As suggested by the example in Figure 2, the function \( k(t) \) is of much shorter duration than \( p(t) \), so that equation (3) is similar to the trace generated by a single impulsive source.

Multiple impulsive sources rapidly deliver a series of short-duration forces over an extended time. Similar to swept frequency sources, crosscorrelation is used to convert each trace to a result that is analogous to a trace from a single impulsive source. The Mini-Sosie is the best known multiple impulsive source; see Barbier and Vialliix (1974) and Barbier et al. (1976) for a review, and Steeples et al. (1986) and Miller et al. (1988) for example applications. A recent contribution is the swept multiple impulsive source described by Park et al. (1996), which combines the ideas of swept frequency and multiple impulsive sources.

Some applications in near-surface seismology involve the use of SH-waves. A suitable source of these waves is a stout timber oriented normal to the direction of an inline offset spread and fixed to the earth’s surface with a hold-down weight or spikes. Traces recorded with blows in opposite directions are subtracted to enhance SH-waves relative to other waves. Two sources of this type are described by Michaels (1998).

\[ \text{Vertical stacking} \]

It is possible to reduce the effect of random noise on a field record by simply adding the records from repeated source efforts with no change in the source location, spread of receivers, or the recording system. This practice is known as \textit{vertical stacking} or \textit{field summing}, and is based on
on the assumption that the source-related portion of each trace will add constructively while random noise will be relatively diminished by destructive interference. An important prerequisite for vertical stacking is the ability of the source to produce the same seismic wavefield from each source effort, a source attribute known as repeatability.

Knapp and Steeples (1986b) and Steeples and Miller (1990) have discussed the repeatability of various sources commonly used in near-surface seismology in terms of the logistics of duplicating both the source mechanics and the ground conditions at the source point. All modern digital seismographs suitable for near-surface seismology are capable of vertical stacking during acquisition to reduce data storage requirements.

A simple statistical analysis of vertical stacking illustrates its dependence on the characteristics of the random noise present in the traces. Consider a seismic experiment in which a highly repeatable source at a fixed location is used to generate a sequence of $n$ traces from a fixed receiver. The trace $A_i(t)$ associated with source effort $i$ may be modeled as the sum of a deterministic function $S(t)$ and a random variable $R_i(t)$:

$$A_i(t) = S(t) + R_i(t).$$

The random variable $R_i(t)$ is of unspecified distribution, but it is stationary for the duration of the trace with zero mean and finite variance $\sigma_i^2$. The function $S(t)$ represents the error-free system response to the source-generated seismic wavefield, and is referred to here as the signal; the remaining portion $R_i(t)$ of the trace is not related to the source and is therefore called the noise.

The size of the signal relative to the noise is given by the signal-to-noise ratio ($S/N$), which is defined for the purposes of this exercise to be the ratio of the signal to the standard deviation of the noise. Thus, the signal-to-noise ratio of trace $i$ is

$$S/N\text{ (trace }i\text{)} = \frac{S(t)}{\sigma_i}.$$  

(5)

and the signal-to-noise ratio of the vertically stacked trace is

$$S/N\text{ (vertical stack)} = \frac{nS(t)}{\sqrt{\sum_{j=1}^{n} \sigma_j^2}},$$

(6)

where the denominator in equation (6) is obtained by assuming that the random variables $R_i(t)$ are mutually independent. The question of whether vertical stacking results in an improved signal-to-noise ratio can be answered by comparison of equations (5) and (6) for a specified set of variances.

One easily investigated possibility is the case in which noise is stationary over the entire time interval required to obtain the $n$ source efforts. In that case, the variance of the noise is the same from trace to trace:

$$\sigma_j^2 = \sigma^2 \quad j = 1, 2, \ldots, n.$$  

(7)

Substitution of assumption (7) into the signal-to-noise equations (5) and (6) shows that vertical stacking gives a signal-to-noise ratio that is larger than the single-trace ratio by a factor of $\sqrt{n}$:

$$\frac{S/N\text{ (vertical stack)}}{S/N\text{ (single trace)}} = \sqrt{n}.$$  

(8)

However, if the noise is not stationary during the acquisition of the $n$ source efforts, then it is possible for vertical stacking to decrease the signal-to-noise ratio relative to the ratio of a single trace. For example, under conditions of sporadic vehicle traffic, it is common for noise bursts to occur so that one trace with excessive noise can drastically reduce the signal-to-noise ratio for the vertically stacked trace. Indeed, from equations (5) and (6), it may be concluded that the signal-to-noise ratio of one of the single traces is always larger than the ratio of the vertically stacked trace unless

$$\frac{1}{n} \sum_{j=1}^{n} \sigma_j^2 < \sigma_i,$$

(9)

for $i = 1, 2, \ldots, n$. Because it is not efficient to carefully monitor noise during most field operations, trace summation schemes have been devised that minimize the deleterious effect of nonstationary variance. Of possible importance to near-surface seismology is diversity stacking, which has the property that the signal-to-noise ratio of the summed trace always increases with $n$ (Gimlin and Smith, 1980). If noise bursts are a problem, and the vertical stacking capability of the seismograph is being utilized during acquisition, then it may be advisable to save the individual field records (one for each source effort) also so that more sophisticated trace summation techniques can be performed during processing.

**Geophones and seismometers**

A geophone or seismometer is an electromechanical transducer that converts ground motion to an analog electrical signal. The term geophone refers to a small lightweight device with a natural frequency of perhaps 10 Hz or greater, whereas a seismometer is a relatively large and heavy device with a natural frequency on the order of 1 Hz. The differences are illustrated by the products of a company that offers a 10-Hz geophone with 20-cm$^3$ volume and 75-g total mass, and a 1-Hz seismometer with 600-cm$^3$ volume and 2-kg total mass.

The design of modern geophones and seismometers is
such that their physical robustness decreases with decreasing natural frequency. As a result, geophones tend to be more suitable for rugged field use whereas seismometers need to be handled with greater care. Practitioners of the surface-wave method may choose seismometers, because the maximum attainable target depth for that method depends on acquisition of relatively low-frequency data (perhaps 1 Hz at the low end), but refraction and reflection seismologists favor geophones because of the need to rapidly deploy and retrieve up to several hundred receivers many times daily. Geophones and seismometers of the type used in near-surface seismology are identical in terms of their theory of operation. Therefore, the following discussion applies to both even though the term geophone is used in the remainder of this section.

**General design and function of a geophone**

A geophone is designed to be securely attached to the solid earth. The dimensions of the volume in which the attachment occurs are small relative to the range of seismic wavelengths. Thus, one speaks of the geophone as being attached to a particle of the earth and moving with that particle, so that the geophone case mimics the particle motion or ground motion during a seismic disturbance. A single-component geophone is constructed with a single transducer and is either a vertical geophone (sensitive to vertical particle motion) or a horizontal geophone (sensitive to particle motion in a specific direction in the horizontal plane). A three-component geophone contains three mutually orthogonal transducers and is intended to provide information on particle motion in three-dimensions.

Most three-component geophones need to be leveled and oriented with respect to a fixed horizontal direction to ensure that one transducer is vertical and the others are in a consistent horizontal direction (i.e., the vertical, radial, transverse configuration). In the Gal’perin three-component configuration, the axis of each transducer makes an angle of approximately 54° with the vertical so that each transducer is affected by gravity in the same way and therefore may be designed identically. The Gal’perin configuration is also somewhat easier to orient in the field.

The choice of geophone depends on the type of wave to be recorded and the direction of wave propagation. Common advice for the recording of body waves is based on the theoretical result that in a homogeneous and isotropic linearly elastic material, simple plane P-waves are longitudinal and simple plane S-waves are transverse. Thus, for example, if the direction of body-wave propagation is approximately vertical, the P-wave should record best on a vertical geophone and the S-wave should record best on a pair of horizontal geophones. Vertical propagation is a common assumption for reflected body waves returning to the surface from depth because refraction in low-velocity near-surface materials tends to bend the upgoing raypath toward the vertical according to Snell’s law.

The SH-component of a plane S-wave should record best on a horizontal geophone with sensor axis oriented perpendicular to the direction of wave propagation; this choice would also be the recommendation for recording Love waves. An experiment to record Rayleigh waves would involve either a horizontal geophone aligned parallel to the direction of wave propagation or a vertical geophone (or both if a complete description of particle motion is desired). Three-component geophones may be used in any terrestrial experiment to effectively record all wave types and to aid in wave identification.

The transducer in nearly all modern geophones is the moving-coil electrodynamic type, and consists of a coil suspended by one or more springs in a magnetic field that is fixed relative to the geophone case. Motion of the earth causes relative motion between the coil and the magnetic field and induces an analog voltage at the geophone terminals according to Faraday’s law. The geophone terminals are connected via wires to a seismograph which amplifies, filters, and converts the analog voltage to a digital signal. In the following discussion, the differential equation governing the relationship between particle motion and the geophone output is presented in the time domain to give the geophone frequency response. An understanding of the geophone frequency response aids the proper design of field experiments, and is essential for the recovery of the original particle motion as either displacement, velocity, or acceleration.

**Geophone equation**

Although the specific design and fabrication of a moving-coil electrodynamic geophone varies with the manufacturer, it is reasonable to represent any geophone of this type as a damped harmonic oscillator. To see that this is indeed the case, a detailed analysis of the vertical geophone model shown in Figure 3 is given in Appendix B. The result of the analysis is the geophone equation which is a second-order linear ordinary differential equation characteristic of damped harmonic oscillation:

$$\frac{d^2 V(t)}{dt^2} + 2 \hbar \omega_0 \frac{dV(t)}{dt} + \omega_0^2 V(t) = C \frac{d^2 \nu_p(t)}{dt^2}, \quad (10)$$

where $V(t)$ is the voltage at the seismograph input, and $\nu_p(t)$ is the vertical velocity of the geophone case (a proxy for vertical particle velocity). The constants $\hbar$, $C$, and $\omega_0 = 2\pi f_0$ are the total damping constant (dimensionless), total transduction constant (V per m/s), and natural frequency ($\omega_0$ in radian/s and $f_0$ in Hz), respectively, and are defined.
in Appendix B in terms of the physical properties of the geophone. It is convenient to view a geophone as a linear shift-invariant system governed by equation (10), with the input and output given by $v_g(t)$ and $V(t)$, respectively. This property is exploited in the following discussion to develop the geophone frequency response.

**Geophone frequency response**

The Fourier integral transform of the geophone equation (10) is

$$\tilde{V}(\omega) + 2h\omega_0 \tilde{V}(\omega) + \omega_0^2 \tilde{V}(\omega) = C(i\omega)^3 \tilde{v}_g(\omega),$$

(11)

where $\tilde{V}(\omega)$ and $\tilde{v}_g(\omega)$ are the Fourier integral transforms of the corresponding time functions. Solving for $\tilde{V}(\omega)$ gives

$$\tilde{V}(\omega) = \frac{C\omega^2 \tilde{v}_g(\omega)}{\omega^2 - \omega_0^2 - i2h\omega_0\omega}.$$  (12)

The geophone frequency response is denoted here by $\tilde{H}(\omega)$, and is given by equation (12) for the situation where the vertical case velocity $v_g(t)$ is an impulse represented by the Dirac delta function $\delta(t)$. If $v_g(t) = \delta(t)$, then $\tilde{v}_g(\omega) = 1$, and equation (12) becomes

$$\tilde{H}(\omega) = \frac{C\omega^2}{\omega^2 - \omega_0^2 - i2h\omega_\omega}.$$  (13)

Notice from equation (13) that $\tilde{H}(\omega)$ is a well-defined complex function for all $\omega$ as long as the total damping constant $h$ is nonzero.

Interpretation of the geophone frequency response (13) is best achieved by computing the amplitude response $|\tilde{H}(\omega)|$ and phase response $\angle \tilde{H}(\omega)$:

$$|\tilde{H}(\omega)| = \frac{C\omega^2}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4h^2\omega^2\omega_0^2}},$$

$$\angle \tilde{H}(\omega) = -\arctan\left(\frac{-2h\omega\omega_0}{\omega^2 - \omega_0^2}\right),$$  (14)

which are shown in Figure 4 plotted as a function of $\omega/\omega_0$ for various values of the total damping constant $h$. The ideal frequency response would have $|\tilde{H}(\omega)| = C$ and $\angle \tilde{H}(\omega) = 0$, because the output voltage would then be a scaled version of the vertical case velocity. However, because Figure 4 indicates that this ideal can only be achieved for $\omega >> \omega_0$, a value of the total damping constant $h$ is sought with the fewest objectionable characteristics. Resonance near $\omega_0$ precludes choosing small $h$, whereas large $h$ is undesirable because rolloff of $|\tilde{H}(\omega)|$ is excessive and $|\tilde{H}(\omega)|$ flattens at unacceptably high frequencies. As a result, geophones are commonly designed with $h = 0.7$ as a compromise. At $h = 0.7$, the amplitude response $|\tilde{H}(\omega)|$ is reasonably flat for frequencies $\omega > 1.5\omega_0$, but it should be kept in mind that the phase response $\angle \tilde{H}(\omega)$ remains nonzero and nonlinear in this frequency range. Nonlinear phase means that differ-

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Schematic illustration of a moving-coil electrodynamic geophone. Structure is axially symmetric about axis AB. Magnetic flux lines are represented by closed loops. Polarity of magnetic pole pieces is indicated by N and S. A horizontally wrapped coil is suspended from a spring and connected to geophone terminals via loose conductors that permit vertical coil movement relative to the case. The spring shown is in an undeformed state.

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Amplitude response (a) and phase response (b) for a geophone as functions of frequency $\omega$ normalized to natural frequency $\omega_0$. Responses are shown for total damping constant $h$ equal to 0.3, 0.7, and 2.0. Amplitude response is normalized to total transduction constant $C$ so that it is dimensionless; phase response, in degrees.
ent frequencies undergo different time shifts, so that \( V(t) \) will be a distorted version of \( v(t) \), even at frequencies where the amplitude response is flat.

**Natural frequency, damping, and total transduction constant**

The natural frequency \( \omega_0 \) can be adjusted by changing the stiffness \( K \) of the spring, or the mass \( m \) of the coil, or both; see equation (B-15):

\[
\omega_0 = \sqrt{\frac{K}{m}}.
\]

Geophones with low natural frequencies involve more massive coils (larger \( m \)) and weaker springs (smaller \( K \)) which accounts for their heft and reduced tolerance to rough handling. Some low-frequency geophones are provided with a clamping mechanism to protect the mechanism during transportation and installation.

The total damping constant \( h \) can be expressed as the sum of an open-circuit damping \( h_o \) and an electromagnetic damping or coil-current damping \( h_{em} \); see equation (B-15):

\[
h_o = \frac{D}{2m\omega_0},
\]

and

\[
h_{em} = \frac{J_{em}}{2m\omega_0(R_C + R_E)},
\]

where \( D \) is the mechanical damping coefficient, \( R_s \) is the resistance of the coil, \( R_e \) is the equivalent resistance of the external circuit connected across the geophone terminals, and \( J_{em} \) is a constant related to the internal electromagnetic properties of the geophone. The equivalent resistance \( R_E \) depends on the nature of the external circuit; see equation (B-8):

\[
R_E = \frac{R_S(R_A + R_L)}{R_S + R_A + R_L},
\]

where \( R_A \) is the seismograph input resistance, \( R_s \) is the shunt resistance, and \( R_L \) is the line resistance between the geophone and the seismograph.

Notice in equation (16) that the open-circuit damping \( h_o \) depends only on internal geophone parameters, whereas the electromagnetic damping \( h_{em} \) depends on internal parameters as well as the external circuit through resistance \( R_E \). If the geophone is disconnected from the recording circuit so that \( R_E \) is infinite, the total damping is given by \( h_o \), which is generally in the range 0.3-0.7, depending on the geophone design. If the geophone is connected to the external circuit, and the seismograph input resistance \( R_A \) is very large compared to the shunt resistance \( R_s \) and the line resistance \( R_L \), then equation (17) indicates that \( R_e \) is approximately equal to \( R_S \), and the electromagnetic damping \( h_{em} \) can thereby be controlled by the shunt resistance.

Finally, as shown in equation (B-15), the total transduction constant \( C \) also depends on the external circuit, but it is reasonable to expect values for \( C \) in the range 1-10 V per m/s.

**Geophone considerations in near-surface seismology**

Effects not included in the geophone theory outlined above can result in departures from an otherwise flat amplitude response at spurious frequencies or parasitic resonance frequencies that are well above the natural frequency. However, geophones are now available with a natural frequency of 10 Hz with flat amplitude response free of spurious or parasitic defects to at least 400 Hz. A geophone with these specifications is consistent with the recommendations of a 1997 workshop on the near-surface seismic reflection method (Steeples et al., 1997), and would also be a good selection for the refraction method. However, the same workshop recommended 100-Hz geophones for seismic frequencies that extend above 400 Hz.

Single geophones are used in most near-surface seismic experiments because geophone arrays (geophones connected in series and/or parallel arrangements to produce a single seismic signal) can adversely affect the recording of high frequencies (Knapp and Steeples, 1986b). However, some practitioners use arrays of not more than three geophones distributed over a distance of 1–2 m oriented parallel to the predominant wind direction to help attenuate wind noise (Steeples and Miller, 1990).

If geophones are connected in series and/or parallel arrangements, it is necessary to evaluate the effect of these interconnections on electromagnetic damping. Unless the electromagnetic damping is adversely affected, series connections in geophone arrays are recommended so that the individual geophone voltages sum (Steeples and Miller, 1990).

It is very important to fully push the geophone spike into firm soil (if possible) with no organic debris (grass, leaves, etc.) between the geophone base and the soil. This practice helps ameliorate the well known high-cut filtering effect associated with geophone-soil coupling (Krohn, 1984). Even if the advice of Krohn is followed, the coupling cannot be perfect because the earth is not perfectly rigid and the geophone does not have zero mass. Others have investigated the problem of geophone-soil coupling both theoretically and experimentally; see Spikes et al. (2001) for a recent experimental study that includes a full summary of previous work.
Recent developments in receiver technology

Attention has been given recently to the speed of receiver deployment with innovations such as mounting an entire geophone spread on a board, piece of channel iron, or other mount to facilitate mass deployment and movement of the spread (Steeples et al., 1999a, b; Schmeissner et al., 2001; Bachrach and Munkerji, 2001), and gimbaled geophones in a land streamer that can be towed into position (van der Veen and Green, 1998; van der Veen et al., 2001). A new velocity sensor designed to operate in the horizontal orientation for detection of SH-waves is described by Sambuelli et al. (2001).

A new variable capacitance micromachined silicon accelerometer has been developed that includes an internal integrated circuit that measures and digitizes the accelerometer response for output (Goldberg et al., 2001). Sensors of this type utilize MEMS technology (micromechanical systems), and are seen as possibly leading to an era of full-wavefield systems in which MEMS digital accelerometers are routinely deployed to record the vector ground motion (Dragoset, 2005). Sensors based on MEMS technology are three-component accelerometers that are insensitive to deployment tilt, but automatically sense the direction of gravity and provide a tilt measurement for component rotation.

In addition, MEMS sensors produce a fully digital 24-bit output that completely eliminates the difficulties associated with analog transmission, and provide a flat amplitude and linear phase response from approximately 1 to 800 Hz (Mougenot, 2004). Because MEMS sensors are likely to be deployed as single units and not as noise-canceling arrays, the petroleum industry is currently debating the merits of the new technology. The impact of MEMS sensors on the methods of near-surface seismology, which normally use single sensors but are governed by a much lower capital expense threshold, remains to be seen.

Traditional and distributed seismographs

A modern seismograph is a field-portable electronic system that amplifies, filters, and digitally records the analog seismic signals produced by the receivers. The evolution of seismograph design has been driven by the demands of increasingly sophisticated seismic imaging techniques and by advances in digital electronics such as the development of microprocessors, compact mass data storage devices, and digital communication systems. At the present time, seismographs used in near-surface seismology may be classified as either traditional seismographs or distributed seismographs, and most near-surface seismic data are collected with traditional seismographs.

A traditional seismograph is a self-contained multi-channel system designed to digitally record on a separate channel the output from each receiver that is active during a given source effort. The seismic signals from the active receivers are delivered to the channel inputs via analog transmission over thinly insulated small-diameter wires (28 AWG) bundled in thick, durable cables with polyurethane jackets. Traditional seismographs are usually (but not necessarily) structured in blocks of 12 channels with the total number of channels selected to match project requirements.

For example, it is common for projects designed to map the bedrock interface at shallow depths with the reflection method to utilize 12- or 24-channel seismographs. As another example, the reflection method applied to map a groundwater aquifer might employ a 48- or 96-channel seismograph. As a final example, the spectral analysis of surface waves (SASW) method was originally designed to use only two channels. Regardless of the number of channels, a traditional seismograph handles at a central location the amplification, filtering, digital conversion, and recording of the analog seismic signals from all active receivers.

A distributed seismograph involves a collection of individual seismographic units dispersed at intervals among the spread of receivers. Each unit handles the amplification, filtering, and digital conversion of the analog signals from a nearby subset of the active receivers. The actual recording of the digital signals may take place within the unit or at a central recording point after digital transmission via radiotelemetry or fiber-optic cables. Distributed seismographs ameliorate problems such as crosstalk and interference from power lines that are associated with analog signal transmission through seismic cables over large distances. Distributed seismographs also promote the use of large numbers of receivers in special projects such as 3D reflection surveys in rough terrain because the receiver spreads are easier to deploy. Although distributed seismographs were introduced in the late 1970s and are now in widespread use in the petroleum industry, they are still relatively uncommon in near-surface seismic applications.

Amplification and filtering

The analog seismic signal produced by each receiver needs to be prepared for input to the analog to digital converter (A/D converter), which is at the heart of a modern seismograph. Although the details of the analog signal preparation vary considerably from one seismograph to the next, the nominal steps are initial amplification (also known as preamplification), filtering, and final amplification with gain control. The initial amplification adjusts the level of the seismic signal to account for the strength of the source relative to the location of the receiver, and is done by a fixed-gain linear amplifier of very low noise and very low distortion. Filtering removes high frequencies that are susceptible to aliasing (see “Bandwidth and sampling,” below) and optionally attenuates noise associated with cer-
tain frequency bands. The final amplification with gain control adjusts the level of the initially amplified and filtered signal so that it is within the input range of the A/D converter.

Gain control known as instantaneous floating point or IFP refers to amplification that applies gain at each sample time based on the signal level at that sample time; the gain is recorded as part of the output of the A/D converter. Gain control is necessary if the dynamic range of the analog signal at the input to the A/D converter exceeds the dynamic range of the converter (see “Dynamic range” below).

Although IFP seismographs are highly regarded and widely used, the most recently introduced seismographs do not include gain-control amplification in the preparation of the analog seismic signal. These newer seismographs use innovative 24-bit sigma-delta A/D converters that have a much larger dynamic range than the 16-bit A/D converters typical of IFP seismographs. Also, because the sigma-delta A/D converters sample at very high sampling rates (e.g., 256 kHz), filtering can be done digitally followed by resampling at the lower sampling rates used to represent each digital trace. The potential advantages of the newer systems include simplicity of design, lower electronic noise, and lower power consumption, because there are fewer analog electronic components.

Bandwidth and sampling

The bandwidth of the amplified and filtered analog seismic signal \( A(t) \) is the interval \( f_l < f < f_u \) on the positive frequency axis within which the power spectrum of the signal has significant magnitude. Because the term significant magnitude is imprecise, many mathematical definitions are used to identify the lower frequency \( f_l \) and upper frequency \( f_u \) of the bandwidth. Rather than explore these definitions, it is sufficient to state that in the context of near-surface seismology, the bandwidth of a seismic signal produced by a surface geophone located in the far field of a near-surface source usually lies well within the 0–1000 Hz range. A reasonable feel for the typical bandwidth of seismic signals in near-surface seismology can be obtained by examining the numerous spectra published in the series of source comparisons cited earlier.

A discrete-time signal \( A(nT) \) is obtained from the amplified and filtered analog seismic signal \( A(t) \) by sampling at a finite number of discrete times \( t = nT \), where \( n = 0, 1, 2, \ldots, N \) and the sampling interval \( T \) is assumed constant. The inverse of the sampling interval

\[
f_s = \frac{1}{T}
\]

is called the sampling rate or sampling frequency. In near-surface seismology, \( T \) and \( f_s \) are usually specified in ms and Hz, respectively. For example, if the sampling interval is 0.5 ms, then the sampling frequency is 2000 Hz. Sampling intervals on modern seismographs are selectable by the operator in a range that can be as broad as 0.02–16.0 ms which corresponds to sampling frequencies of 62.5–50 000 Hz.

A well-known result from linear system theory known as the sampling theorem states that a strictly band-limited analog signal \( A(t) \) can be recovered from the corresponding discrete-time signal \( A(nT) \) if the sampling frequency \( f_s \) is at least twice the frequency \( f_u \) of upper end of the bandwidth of the signal:

\[
f_s \geq 2f_u. \quad (19)
\]

The undesirable condition \( f_s < 2f_u \) is referred to as aliasing, and causes signal content at frequencies above \( f_u/2 \) to be improperly represented as signal content at lower frequencies; the frequency \( f_u/2 \) is called the Nyquist frequency or folding frequency in near-surface seismology (but Nyquist frequency refers to the sampling frequency in some other fields). To prevent aliasing of a band-limited signal, either the sampling frequency needs to be increased or the analog signal must be low-pass filtered so that condition (19) is met. Analog low-pass filters aimed at fulfilling condition (19) are known as alias filters or antialias filters, and are found on many seismographs prior to the conversion of the signal to digital form.

Quantization

A digital signal is obtained from a discrete-time signal by quantizing the sampled values. Recall that \( A(nT) \) represents the sampled values of the amplified and filtered analog seismic signal \( A(t) \), and suppose that the sampled values fall within the range \([-A_{\max}, A_{\max}]\), where \( A_{\max} \) is a finite positive number. Quantization is the process of assigning a sampled value \( A(nT) \) to one level \( A_m \) of a finite number \( M \) of levels representing the range \([-A_{\max}, A_{\max}]\):

\[
-A_{\max} < A_1 < A_2 < \ldots < A_m < \ldots < A_M \leq A_{\max}. \quad (20)
\]

The step size of a quantizer is the difference between two adjacent levels and is given by

\[
\Delta_m = A_{m+1} - A_m \quad 1 \leq m \leq M - 1. \quad (21)
\]

In a uniform quantizer, the step size does not depend on \( m \) and may be represented by \( \Delta \). Figure 5 shows an example of uniform quantizer where the sampled value \( A(nT) \) is assigned level \( A_m \) if the following rule is satisfied:

\[
\left( A_m - \frac{\Delta}{2} \right) \leq A(nT) < \left( A_m + \frac{\Delta}{2} \right). \quad (22)
\]

The quantization error \( E \) is the difference between the sampled value \( A(nT) \) and the level \( A_m \) chosen to represent it:
The quantization error for an assignment rule such as equation (22) cannot exceed $\Delta/2$, which implies that the maximum quantization error can be reduced by reducing the step size. Reducing the step size for a given $A_{\text{max}}$ can be accomplished by increasing the number $M$ of levels. Also, notice that for a uniform quantizer, the maximum quantization error expressed as a percentage of the sampled value $A(nT)$ decreases with increasing $A(nT)$.

Each amplitude level $A_m$ is represented by a code that is amenable to digital electronic systems. Binary codes are commonly chosen because the binary digits 0 and 1 (also known as bits) are resistant to electronic noise in transmission systems. The number $K$ of unique code words in a binary code depends on the number of bits $N_b$ in a binary word:

$$K = 2^{N_b}. \quad (24)$$

For example, if $N_b = 8$ in a binary code, then the number $K$ of unique code words is 256. Clearly, the number of required bits $N_b$ depends on the number of levels available in an A/D converter. A larger $M$, which requires a larger $N_b$, corresponds to a reduced step size (and hence reduced quantization error as mentioned previously) and/or an expanded input range limit $A_{\text{max}}$ (and hence expanded dynamic range as explained below).

Once the quantizer has assigned a level $A_m$ to the sampled value $A(nT)$, the final step is to represent $A(nT)$ by the binary word $B_m$ that uniquely represents $A_m$. The A/D conversion of an amplified and filtered analog seismic signal $A(t)$ of duration $NT$ can therefore be summarized as follows:

$$A(t) \rightarrow A(nT) \rightarrow A_m(nT) \rightarrow B_m(nT) \quad (25)$$

where the specific characteristics of the quantization and coding steps depend on the many different digital seismograph designs that have been developed over the years.

**Dynamic range**

Dynamic range is a general term that refers to the range of positive values that can be accurately measured and recorded by a measuring system. For example, the dynamic range for a meter bar marked in increments of 1 mm, and used to measure lengths, is reasonably given as 0.2–1000 mm, where 0.2 mm is considered the smallest length that can be judged to be different from zero with the unaided eye. If $S_{\text{min}} > 0$ and $S_{\text{max}} > S_{\text{min}}$ are the minimum and maximum values of the dynamic range, respectively, then it is common to express the dynamic range as the ratio $S_{\text{max}}/S_{\text{min}}$, where the ratio is converted to the decibel scale (dB) by the following standard formula:

$$\text{dynamic range} \equiv 20 \log_{10} \left( \frac{S_{\text{max}}}{S_{\text{min}}} \right) \text{ dB}. \quad (26)$$

Applying equation (26) to the example of the meter bar gives a dynamic range of 74 dB. It is useful to recognize that because $20\log_{10}(2)$ is 6.02 dB, every 6 dB increase in the dynamic range corresponds very closely to an increase by a factor of two in the $S_{\text{max}}/S_{\text{min}}$ ratio. Thus, the dynamic range 74 dB corresponds to a ratio of maximum to minimum measurements greater than $2^{12}$.

The dynamic range of a seismograph is measured and reported in a variety of ways and depends on numerous factors. Perhaps the most useful general definition is the input dynamic range defined by Pieuchot (1984, 326), and given in terms of the decibel scale as follows:

$$\text{input dynamic range} \equiv 20 \log_{10} \left( \frac{S_{\text{max}}}{N_{\text{input}}} \right) \text{ dB} \quad (27)$$

where $S_{\text{max}}$ is the maximum input signal, and $N_{\text{input}}$ is the system noise which masks any smaller input signal. Both $S_{\text{max}}$ and $N_{\text{input}}$ are measured in volts at the input terminals of the seismograph at a particular gain setting for the ini-
A/D converter as follows:

For example, suppose the maximum positive input voltage at zero preamplifier gain is 1.4 V, and the system noise is given as 0.20 $\mu$V at a preamplifier gain of 36 dB. Assuming that the 1.4-V maximum input is prescribed by a device after the preamplifier, the value of $S_{\text{max}}$ that can be input to the 36-dB preamplifier is estimated to be 0.022 V (i.e., the 1.4-V maximum divided by the 2$^6$ gain of the preamplifier set at 36 dB). Thus, with $S_{\text{max}}$ given by 0.022 V, definition (27) gives an estimate of 101 dB for the input dynamic range of the seismograph at the 36-dB preamplifier setting. If the maximum voltage of the seismic signal is aligned with the maximum allowable input of the seismograph (0.022 V), this dynamic range is sufficient to record the 84-dB seismic signal estimated by Pieuchot (1984, 54–55) for the most demanding case of a near-source receiver.

Sometimes the dynamic range of a fixed-gain seismograph is estimated from the number of bits produced by the A/D converter as follows:

$$\text{estimated dynamic range} = 6(N_b - 1), \quad (28)$$

where $N_b$ is the number of bits in the binary code. This formula stems from the idea that if one bit is reserved for the algebraic sign, then the smallest through largest positive signal amplitudes are represented by the range of binary numbers associated with the remaining bits. Thus, a fixed-gain seismograph with a 16-bit A/D converter, where 1 bit represents algebraic sign, has an estimated dynamic range of 90 dB. The primary disadvantage of the dynamic range estimate in equation (28) is that it ignores the effect of system noise.

**Refractive Method**

The refraction method is the oldest of the near-surface seismic methods and continues to be widely used for mapping the geometry of shallow geologic interfaces in a variety of applications. Accurately timed first arrivals on field records are the basic data, and correct identification of the waves responsible for the first arrivals is of critical importance. This section focuses on the simplest possible two-dimensional case where a single shallow interface separates surficial material from an underlying material of contrasting physical properties. If the seismic velocity of the surficial layer is less than the velocity of the underlying layer, then beyond a certain distance from the source, the first arrival will be a head wave generated at the shallow interface. At lesser source-receiver distances, the first arrival is generally expected to be a body wave propagating in the surficial layer. First-arrival times at a series of receivers deployed along a line of sufficient length between two surface sources usually provide adequate data for estimating depth to the interface. The determination of depth to bedrock beneath an unconsolidated overburden is the classic application of the refraction method.

A refraction survey can be carried out with field equipment that is relatively simple. A fixed-gain digital seismograph with at least 12 channels, a selection of acquisition filters, and the capability to vertically stack repeated source efforts would probably be considered adequate by most practitioners. Geophones should be suitable for rugged field work with a clean passband of perhaps 10–200 Hz. Appropriate sources are the single impulsive type such as a sledge hammer, a small explosive, or a trailer-mounted accelerated weight drop. The required source energy increases as the interface depth increases because greater source-receiver distances are needed to observe the head waves.

The theory of the refraction method is based on the tracing of critically incident rays, and is therefore governed by the assumptions implicit in geometric ray theory. A large number of interpretation methods have been published over the years, but they are all closely related through their common dependence on Snell's law. Although a fairly sophisticated interpretation can be carried out entirely with pencil, graph paper, and hand calculations, it is also possible to apply state-of-the-art computer-based inversion routines that are available both commercially and in the public domain. Indeed, modern seismographs designed for near-surface seismic applications are now bundled with refraction interpretation software. None of this advanced software, however, relieves the seismologist from the responsibility of making correct identifications of the waves represented on the traveltime curves.

Outside of the many textbooks on applied geophysics, some of the important general references for the near-surface refraction method are Redpath (1973), Palmer (1980, 1981, 1986), Haeni (1988), and Lankston (1989, 1990). The volume of papers edited by Musgrave (1967) is derived mostly from refraction applications in the petroleum industry, but contains a history of early refraction work and also an extensive bibliography. The American Society for Testing and Materials (ASTM) has published standard D5777-00 for the refraction method (“Standard Guide for Using the Seismic Refraction Method for Subsurface Investigations”). Published case histories also constitute a valuable source of information (see supplementary references at the end of this chapter).

**Single plane dipping interface and the slope-intercept method**

The **slope-intercept method** of interpretation for a single plane dipping interface illustrates many of the basic features of the refraction method. Consider a plane dipping interface separating two materials of contrasting seismic
velocity as shown in Figure 6. Suppose the velocity $V_1$ of the upper material is less than the velocity $V_2$ of the lower material, and that receivers are distributed along the surface on a straight line between source locations $A$ and $B$. The horizontal distances from $A$ and $B$ to a given receiver $G$ are $x$ and $y$, respectively, where $x + y = d$, and constant $d$ is the total length of the profile. The traveltime equations for the direct waves that travel from $A$ and $B$ to $G$ are simply the horizontal distances divided by the velocity of the upper layer:

$$T_{dw}(A \rightarrow G) = \frac{x}{V_1}$$

and

$$T_{dw}(B \rightarrow G) = \frac{y}{V_1},$$  \hspace{1cm} (29)

The corresponding traveltime equations for the head waves that travel from $A$ and $B$ to $G$ are derived by tracing the critically incident rays from source to receiver, and then putting the resulting traveltime expressions in a standard form:

$$T_{hw}(A \rightarrow G) = \frac{\sin(\theta_c - \xi)}{V_1} x + \frac{2z_A \cos \theta_c}{V_1} \text{ (updip)}$$

and

$$T_{hw}(B \rightarrow G) = \frac{\sin(\theta_c + \xi)}{V_1} y + \frac{2z_B \cos \theta_c}{V_1} \text{ (downdip)},$$  \hspace{1cm} (30)

where $\xi$ is the dip angle and $\theta_c$ is the critical angle of incidence:

$$\theta_c = \arcsin \left( \frac{V_1}{V_2} \right).$$  \hspace{1cm} (31)

Parameters $z_A$ and $z_B$ are the depths measured normal to the interface beneath source locations $A$ and $B$, respectively. Because of reciprocity (i.e., the principle that the traveltime does not change if the source and receiver switch positions), the direct-wave traveltime $T_{dw}(A \rightarrow B)$ must equal $T_{dw}(B \rightarrow A)$, and the head-wave traveltime $T_{hw}(A \rightarrow B)$ must equal $T_{hw}(B \rightarrow A)$; these requirements of equations (29) and (30) are easily checked by substituting $d$ for $x$ and $y$ and noting that $z_A = z_B + d \sin \xi$.

Equations (29) and (30) are observable linear functions of $x$ and $y$ with measurable slopes and intercepts that provide complete characterization of the single plane dipping interface. Specifically, equation (29) indicates that the upper layer velocity $V_1$ may be estimated by the inverse of the slope of the direct-wave traveltime equation. Furthermore, inspection of equation (30) shows that the other unknown subsurface parameters are related by the following formulas to the head-wave traveltime slopes and intercepts:

$$\begin{align*}
  m_A &= \frac{\sin(\theta_c - \xi)}{V_1} \\
  m_B &= \frac{\sin(\theta_c + \xi)}{V_1} \text{ (slopes)}
\end{align*}$$

$$\begin{align*}
  b_A &= \frac{2z_A \cos \theta_c}{V_1} \\
  b_B &= \frac{2z_B \cos \theta_c}{V_1} \text{ (intercepts)}
\end{align*}$$

The expressions in equation (32) can be solved for the critical angle $\theta_c$ and dip angle $\xi$ in terms of the slopes:

$$\theta_c = \frac{\arcsin(V_1 m_B) + \arcsin(V_1 m_A)}{2}$$

and

$$\xi = \frac{\arcsin(V_1 m_B) - \arcsin(V_1 m_A)}{2}. $$  \hspace{1cm} (34)

Similarly, the expressions in equation (33) can be solved for the depths $z_A$ and $z_B$ in terms of the intercepts:

$$z_A = \frac{V_1 b_A}{2 \cos \theta_c}$$

and

$$z_B = \frac{V_1 b_B}{2 \cos \theta_c}. $$  \hspace{1cm} (35)
Velocity \( V_2 \) of the lower layer may be obtained from equation (31) by substituting the critical angle of incidence estimated by application of equation (34). Thus, if the slopes and intercepts of the traveltime curves [equations (29) and (30)] are determined by direct-wave and head-wave traveltime measurements made between source points \( A \) and \( B \), it is straightforward to construct a complete map of the single plane dipping interface.

Notice that the single plane dipping interface problem may be viewed as having four unknowns (for example, \( V_1 \), \( V_2 \), \( z_A \), and \( z_B \)), and four independent equations. The independent equations are one direct-wave traveltime equation (29), both head-wave traveltime equations (30), and Snell’s law (31) for the critical angle. Clearly, unless the interface is horizontal so that \( z_A = z_B \) and there are only three unknowns, it is not possible to obtain a unique solution without both the updip and downdip head-wave traveltime equations. The seismic refraction method is therefore always characterized by forward and reverse data acquisition, corresponding to source locations at each end of a linear spread.

**Single undulating interface and the reciprocal method**

The reciprocal method is an elegant solution to the problem of mapping an undulating interface that separates two materials of contrasting seismic velocity beneath an undulating surface: the description of the method by Hawkins (1961) is adapted for presentation here. In the reciprocal method, quantities known as the *time-depth* \( t_G \) and *velocity analysis function* \( t_v \) are defined for a receiver \( G \) in terms of measured head-wave traveltimes:

\[
\begin{align*}
  t_G & = \frac{T_{h1}(A \rightarrow G) + T_{h2}(B \rightarrow G) - T_{h1}(A \rightarrow B)}{2} \\
  t_v & = \frac{T_{h1}(A \rightarrow G) - T_{h2}(B \rightarrow G) + T_{h1}(A \rightarrow B)}{2}
\end{align*}
\]

(36)

where \( A \) and \( B \) are the locations of two surface sources, and receiver \( G \) is located on the surface between \( A \) and \( B \) (see Figure 7). The time-depth and velocity analysis function are significant because \( t_G \) can be related to the interface depth \( h \) beneath \( G \), and \( t_v \) can be used to infer the velocity \( V_2 \) of the lower layer.

To find the relationship between \( t_G \) and \( h \), the head-wave traveltimes in equation (36) are expressed in terms of the raypath segments defined in Figure 7 to give the following result:

\[
t_G = \frac{1}{2} \left( \frac{DG}{V_1} + \frac{FG}{V_1} - \frac{DF}{V_2} \right). \tag{37}
\]

If \( DF \) is a straight line representing a plane segment of the interface, then triangle \( GDF \) is an isosceles triangle, and equation (37) simplifies as follows:

\[
t_G = \frac{DG}{V_1} - \frac{DE}{V_2}. \tag{38}
\]

Using simple trigonometric relationships evident in Figure 7, and Snell’s law \( \sin \theta_c = V_1/V_2 \) for the critical angle \( \theta_c \), equation (38) can be rewritten to give a linear relationship between \( t_G \) and the depth \( h \), where \( h \) is measured normal to the interface beneath receiver \( G \):

\[
t_G = \frac{h \cos \theta_c}{V_1}. \tag{39}
\]

Equation (39) may be solved for \( h \), and \( \cos \theta_c \) is written in terms of the velocities to give

\[
h = \frac{V_1 V_2}{\sqrt{V_2^2 - V_1^2}} t_G = C_f t_G, \tag{40}
\]

where the *conversion factor* \( C_f \) is defined as follows:

\[
C_f = \frac{V_1 V_2}{\sqrt{V_2^2 - V_1^2}}. \tag{41}
\]

The depth \( h \) is computed directly from the time-depth \( t_G \) assuming that \( C_f \) is known. Once \( h \) has been computed for a number of receivers along the profile between \( A \) and \( B \), the interface is drawn as the envelope of the set of arcs of varying radius \( h \) centered on each receiver position.

**Figure 7.** Raypaths for analysis of time depth \( t_G \) and velocity analysis function \( t_v \) in the reciprocal method. Sources at \( A \) and \( B \), and receiver at \( G \). Head-wave arrivals reach receiver \( G \) by traveling raypaths \( ACDG \) and \( BHFG \). Line segment \( GE \) of length \( h \) is normal to subsurface interface. Variable \( x \) is the distance from \( A \) to \( G \) along the surface. Variable \( x_7 \) is the distance from \( C \) to \( D \) to \( E \) along the subsurface interface. Angle \( \theta_c \) is the critical angle of incidence. All dashed lines are normal to the subsurface interface. Velocities above and below the interface are \( V_1 \) and \( V_2 \), respectively.
Equation (41) indicates that the conversion factor \( C_f \) is dependent on velocities \( V_1 \) and \( V_2 \). A value for \( V_1 \) may be obtained from the inverse slope of direct-wave travel times; see equation (29). The problem of determining \( V_2 \) can be approached by using the velocity analysis function. Rewriting the definition of \( t_v \) in terms of raypath segments shows that \( t_v \) is the head-wave traveltime from source location \( A \) on the surface to position \( E \) on the interface:

\[
t_v = \frac{AC}{V_1} + \frac{CDE}{V_2}. \tag{42}
\]

Because \( \frac{AC}{V_1} \) is a constant, \( t_v \) may be regarded as a function of distance \( x_p \), where \( x_p \) is the distance measured along \( CDE \) and its extension on the interface. Thus, writing \( x_p \) for \( CDE \) in equation (42), the velocity \( V_2 \) is given by the inverse of the derivative of \( t_v \) with respect to \( x_p \):

\[
V_2 = \left( \frac{dt_v}{dx_p} \right)^{-1}. \tag{43}
\]

Of course, the distance \( x_p \) is unavailable initially, so \( t_v \) is plotted as a function of the distance \( x \) measured from \( A \) to \( G \) along the surface. The inverse slope of this plot yields an approximate \( V_2 \) (which becomes exact if the surface and interface are parallel):

\[
V_2 \approx \left( \frac{dt_v}{dx} \right)^{-1}. \tag{44}
\]

This approximate velocity yields an approximate \( C_f \) and a set of approximate \( h \)-values computed from equation (40). The resulting approximate interface may then be used to estimate the distance \( x_p \). Replotting \( t_v \) as a function of the \( x_p \) estimate leads to a refined \( V_2 \) estimate. Thus, if necessary, an iterative approach can be used to determine both \( V_2 \) and the interface.

The conversion factor \( C_f \) can also be determined directly from equation (40) if the interface depth \( h \) is known at one or more locations along the profile (perhaps from borehole data), and the corresponding time-depth \( t_G \) is also known at those locations. This alternate procedure of determining \( C_f \) can directly account for lateral velocity variation and provide quality control through independent depth information.

**Single undulating interface and the generalized reciprocal method**

The *generalized reciprocal method* (GRM) was developed by Palmer (1980, 1981), and includes the reciprocal method as a special case. Like the reciprocal method, the GRM is used to map an undulating interface that separates two materials of contrasting seismic velocity beneath an undulating surface. However, the GRM differs from the reciprocal method in that it generalizes the definitions for the time-depth \( t_G \) and the velocity analysis function \( t_v \):

\[
t_G = \frac{T_{h-o}(A \rightarrow Y) + T_{h-o}(B \rightarrow X) - T_{h-o}(A \rightarrow B)}{2}
\]

and

\[
t_v = \frac{T_{h-o}(A \rightarrow Y) - T_{h-o}(B \rightarrow X) + T_{h-o}(A \rightarrow B)}{2}, \tag{45}
\]

where \( A \) and \( B \) are the locations of two surface sources, \( X \) and \( Y \) are two receivers on the surface between \( A \) and \( B \), point \( G \) is midway between \( X \) and \( Y \), and \( x \) is distance measured along the surface from \( A \) to \( G \) (see Figure 8). Notice that the GRM definitions for the time-depth and the velocity analysis function reduce to the reciprocal method definitions if receivers \( X \) and \( Y \) occupy the same position as point \( G \) [equation (36)]. Thus, as stated above, the reciprocal method is a special case of the GRM.

The following analysis shows that the GRM time-depth \( t_G \) and velocity analysis function \( t_v \) are related to the interface depth \( h \) and velocity \( V_2 \), respectively, in ways that are identical to the reciprocal method. Start by expressing the head-wave traveltimes in equation (45) in terms of the

![Figure 8. Raypaths for the analysis of time depth \( t_G \) and velocity analysis function \( t_v \) in the generalized reciprocal method. Sources at \( A \) and \( B \), and receivers at \( X \) and \( Y \). Point \( G \) on the surface is midway between \( X \) and \( Y \). Head-wave arrivals reach receiver \( Y \) from source \( A \) by traveling raypath \( AHET \). Head-wave arrivals reach receiver \( X \) from source \( B \) by traveling raypath \( BKCX \). Line segment \( GD \) of length \( h \) is normal to subsurface interface. Constants \( h_x \) and \( h_y \) are the lengths of the line segments drawn normal to the extension of line segment \( CE \) from \( X \) and \( Y \), respectively. Variable \( x \) is the distance from \( A \) to \( G \) along the surface. Angle \( \xi \) is the angle between line segments \( XY \) and \( CE \). Angle \( \theta_1 \) is the critical angle of incidence. All dashed lines are normal to subsurface interface. Velocities above and below the interface are \( V_1 \) and \( V_2 \), respectively.](image)
raypath segments defined in Figure 8, so that \( t_G \) and \( t_r \) may be written as follows:

\[
 t_G = \frac{1}{2} \left( \frac{C\bar{X}}{V_1} + \frac{EY - C\bar{E}}{V_2} - X\bar{Y} \left( \frac{dt_x}{dx} \right) \right)
\]

and

\[
 t_r = \frac{AH}{V_1} + \frac{HC}{V_2} + \frac{C\bar{E} - E\bar{Y}}{2V_2}.
\]

Next assume that the geometric figure \( XCEY \) may be approximated by a quadrilateral. Under this assumption, which is equivalent to assuming that portions of the surface and interface are approximately planar, straightforward trigonometry may be applied to equation (46) to give

\[
 t_G = \frac{1}{2} \left( \frac{h_x + h_y}{V_1} + \frac{\bar{X}\bar{Y} \cos \xi}{V_2} - \bar{X}\bar{Y} \left( \frac{dt_x}{dx} \right) \right)
\]

\[
 t_r = \frac{AH}{V_1} + \frac{HD}{V_2} + \frac{\bar{X}\bar{Y} \sin \xi}{2V_1 \cos \theta_e}.
\]

where \( \xi \) is the angle between quadrilateral sides \( \bar{X}\bar{Y} \) and \( CE \). Although \( t_r \) is not shown in equation (47) as an explicit function of the distance \( x \), it is possible to compute \( dt_r/dx \) using the definition for the derivative:

\[
 \frac{dt_x}{dx} = \lim_{\Delta x \to 0} \frac{\Delta t_x}{\Delta x} = \frac{\Delta HD}{\Delta x} \frac{V_2}{V_1}
\]

\[
 = \left( \frac{\Delta \cos \xi}{\Delta x} \right) = \frac{\cos \xi}{\bar{X} \bar{Y}},
\]

where \( \Delta x \) is an infinitesimal increment in distance \( x \), and \( \Delta t_x \) is the corresponding increment in the velocity analysis function. Rewriting equation (48) gives \( V_2 \) in terms of the inverse of the derivative \( dt_x/dx \):

\[
 V_2 = \cos \xi \left( \frac{dt_x}{dx} \right)^{-1}.
\]

Equations (48) and (49) indicate that the inverse slope of \( t_r \) overestimates the velocity \( V_2 \) by the factor \( 1/\cos \xi \), but the error is within 3.5% of the true velocity if the angle \( \xi \) is less than 15°. The \( \cos \xi \) term in equation (49) can be viewed as a correction factor that is missing in the approximation expressed by equation (44) for the reciprocal method.

The derivative (48) may be substituted into equation (47) to give the GRM relationship between time-depth and interface depth:

\[
 t_G = \frac{h \cos \theta_e}{V_1},
\]

where the depth \( h = (h_x + h_y)/2 \) is measured normal to the interface beneath \( G \). Solving equation (50) for \( h \), and writing \( \cos \theta_e \) in terms of \( V_1 \) and \( V_2 \), gives the following result:

\[
 h = C_f t_G,
\]

where the conversion factor \( C_f \) is defined by \( C_f = V_1 V_2 / \sqrt{V_1^2 - V_2^2} \). Equations (50) and (51) are identical to equations (39) and (40) for the reciprocal method. Thus, as in the reciprocal method, the depths \( h \) are estimated according to equation (51) using the measured time-depths for a number of points on the surface, and the interface is drawn as the envelope of the set of arcs centered on the surface points, with the arc radii given by the estimated depths.

Although the reciprocal method and the GRM lead to the same basic relationship between time depth and interface depth, the GRM analysis can be carried out for a variety of \( X\bar{Y} \)-separations. Palmer (1980, 1981) argues that the best \( X\bar{Y} \)-separation (denoted here by \( X\bar{Y}_o \) and called the optimum \( X\bar{Y} \)-separation) is the separation that corresponds to coincidence of points \( C \) and \( E \) in Figure 8. Coincidence of these points eliminates the planar element assumption along the interface, and presumably leads to a more detailed description of the interface. Using the results of extensive modeling as a guide, Palmer (1980, 1981) gives procedures for deducing \( X\bar{Y}_o \) from plots of time-depth and the velocity analysis function for a range of \( X\bar{Y} \)-separations. If the value of \( X\bar{Y}_o \) inferred from the plots does not agree with the optimum \( X\bar{Y} \)-separation computed from the interpreted model, then this inconsistency may indicate the presence of undetected layers (i.e., layers not represented as first arrivals in the head-wave traveltime data). Palmer (1980, 1981) advocates the use of an average velocity approach to estimate the depth to an interface even though undetected layers may occur above it. Recently, some authors have critically examined the difficulties associated with determining the optimum \( X\bar{Y} \)-separation (e.g., Leung, 2003).

**Single undulating interface and inversion for delay times**

Another traditional approach to the interpretation of seismic refraction data is the *delay-time method* originally proposed by Gardner (1939). A modern version using discrete linear inverse methods to estimate delay times is demonstrated by Michaels (1995) and is summarized here. As in the description of the reciprocal and generalized reciprocal methods, consider an undulating interface that separates two materials of contrasting seismic velocity beneath an undulating surface (Figure 9). Further consider a head wave that travels between surface positions \( P_i \) and \( P_j \) along the path \( P_i CD F_j \). The head-wave traveltime can be written as a linear function of the distance \( x_{kl} \) measured from \( I_k \) to \( I_j \) along the interface:
\[ T_{hw}(P_k \rightarrow P_l) = \frac{h_k}{V_1} + \Delta t(P_k) + \Delta t(P_l), \]  

(52)

where \( \Delta t(P_k) \) and \( \Delta t(P_l) \) are the delay times defined in terms of traveltimes along certain raypath segments:

\[ \Delta t(P_k) = \frac{P_k C}{V_1} - \frac{I_k C}{V_2}, \]

and

\[ \Delta t(P_l) = \frac{P_l D}{V_1} - \frac{I_l D}{V_2}. \]

(53)

Notice that the delay times approach zero if \( P_k \) and \( P_l \) are allowed to approach their counterparts \( I_k \) and \( I_l \) on the interface (\( C \) must approach \( I_k \) and \( D \) must approach \( I_l \) to maintain the critical angle of incidence). Thus, the delay times represent the “delay” associated with the raypath segments through the upper layer.

Next, introduce the assumption that the interface segment beneath any source or receiver is planar over a large enough area that the delay time is not dependent on the azimuth of the head-wave raypath. Notice that this assumption does not require a horizontal interface but does imply that triangles \( AP_k C \) and \( DP_l F \) are isosceles triangles, and that the planar bases of the cones formed by rotating triangle \( AP_k C \) about \( P_k I_k \), and triangle \( DP_l F \) about \( P_l I_l \) both lie on the interface. Using this somewhat simplified geometry, and Snell’s law \( \sin \theta = \frac{V_2}{V_1} \) for the critical angle, it is straightforward to deduce the following expressions for the delay times:

\[ \Delta t(P_k) = \frac{h_k \cos \theta_c}{V_1} \]

and

\[ \Delta t(P_l) = \frac{h_l \cos \theta_c}{V_1}. \]

(54)

Solving equation (54) for \( h \), and writing \( \cos \theta_c \) in terms of \( V_1 \) and \( V_2 \), gives the familiar result:

\[ h_k = C_f \Delta t(P_k) \]

and

\[ h_l = C_f \Delta t(P_l). \]

(55)

where the conversion factor \( C_f \) is defined by \( C_f = \frac{V_1 V_2}{V_2} \). Comparison of equation (55) with equations (40) and (51) shows that the delay times are related to the interface depths in precisely the same way as the time-depths in the reciprocal and generalized reciprocal methods. If the conversion factor and the delay times are known, then result (55) gives the interface depths.

To make further progress, let \( x_{ij} \) be the distance measured from \( P_k \) to \( P_l \) along the surface, and assume that the surface and interface topography is sufficiently subdued that \( x_{ij} \) may be approximated by \( x_{ij} \) in equation (52):

\[ T_{hw}(P_k \rightarrow P_l) = \frac{x_{ij}}{V_2} + \Delta t(P_k) + \Delta t(P_l). \]

(56)

The velocity \( V_2 \) and the delay times in equation (56) are viewed as unknowns, whereas the head-wave traveltime is considered known by measurement. Because there are three unknowns and only one constraining equation, it is necessary to acquire additional data in the form of head-wave traveltimes measured along different interface raypaths but involving the same delay times.

Consider a refraction survey with multiple sources and receivers (not necessarily on the same linear profile). Using the head-wave traveltime data of the survey, suppose \( N \) equations of the form (56) can be written, and further suppose that these equations involve \( N_p < N \) distinct surface positions serving as end points for head-wave travel paths. Because each distinct surface position is associated with a unique delay time, the linear system of \( N \) equations involves \( N_p + 1 \leq N \) unknowns, where the unknowns are the \( N_p \) delay times and the velocity \( V_2 \). The system of equations can be written in matrix form as follows:

\[ T_{hw} = Km, \]

(57)

where \( T_{hw} \) is an \( N \times 1 \) column vector of observed head-wave traveltimes, parameter vector \( m \) is an \((N_p + 1) \times 1\) column vector listing all of the unknown delay times plus the slowness \( 1/V_2 \), and \( K \) is an \( N \times (N_p + 1) \) matrix containing the distances \( x_{ij} \) in the first column, with zeros and ones indicating the participating delay times in the remaining columns.

As a simple example, consider a reversed inline spread as shown in Figure 10. The details of equation (57) for this example are as follows:
Other considerations

All of the traditional interpretation procedures used in the refraction method assume that the head wave from a particular interface has been correctly identified on the field records. This aspect of the interpretation is fundamentally important, and is carried out primarily by inspecting the first-arrival times for crossover points marked by changes in apparent velocity. Identification of the crossover from the direct wave to the first head-wave is often reasonably certain because it is usually characterized by a very abrupt increase in apparent velocity with increasing offset, and also by a waveform change.

However, additional crossovers at greater source-receiver distances are commonly observed and are more troublesome to interpret. These additional crossovers can be caused by any of the following circumstances: (1) a crossover to a new head-wave arrival from a deeper interface; (2) a lateral change in the dip of an interface; (3) a lateral change in the velocity of the material beneath the interface. In order to distinguish between a deeper interface and a lateral change in dip and/or velocity, many authors recommend multiple source locations with differing inline offsets. For example, if the source is moved to greater offset for an inline profile, a deeper interface will be revealed by a comparable lateral shift in the crossover point toward the new source location. However, if the crossover is the result of a lateral change in dip and/or velocity, it will remain fixed in absolute location. See Lankston (1990) for illustrations of these examples.

Another very common problem is lack of adequate forward and reverse overlap of head-wave first arrivals from the same interface. As an extreme example, it is possible for head-wave first arrivals from the same interface to be accurately timed in both the forward and reverse directions, but not have any receivers that recorded the head wave in both directions. Thus, in the reciprocal or generalized reciprocal methods of interpretation, it may not be possible to compute time-depths at the desired number of surface locations. The solution to this problem is to compute phantom arrival times using data acquired for source positions at greater inline offsets (see Redpath, 1973; Lankston and Lankston, 1986; and Lankston, 1990 for a full explanation).

It is possible for a head wave to be generated at an interface but not be observed as a first arrival. Because head-waves arriving after the first motion are rarely distinctive enough to reliably recognize and time, the layer beneath this type of interface is hidden from the interpreter. A layer that constitutes a velocity inversion also will not be represented by a head wave from its upper surface, because head-wave generation requires a velocity increase. The presence of hidden layers and/or velocity inversions between the surface and the interface of interest can cause significant errors in the depth calculations to that interface; this difficulty is known as the undetected layer problem. As already mentioned, Palmer (1980) has suggested using an average velocity approach with the generalized reciprocal method to alleviate depth errors of this type. The use of postcritical reflections to constrain the undetected layer problem is discussed by Sain and Reddy (1997).

Multiple interfaces and more general subsurface models

The reciprocal method and generalized reciprocal method can be applied in the case of multiple undulating layers (Hawkins, 1961; Palmer, 1980, 1981). The interpretation is carried out sequentially for each interface, beginning with the uppermost interface, or more commonly by using an average velocity to represent the layers overlaying the interface of interest. Alternatively, fast raytracing algo-
rithms and numerical eikonal solvers have led to the implementation of inversion schemes for traveltimes of waves traversing a general two-dimensionally heterogeneous subsurface (e.g., Zelt and Smith, 1992; Lanz et al., 1998). An initial velocity model is specified using some parameterization of the subsurface. The parameterization may include undulating interfaces marking discontinuous velocities, and continuous velocity variations may occur in the layers between interfaces.

Theoretical traveltimes derived from intermediate subsurface models are compared with observed traveltimes, and the current velocity model is adjusted based on the mismatch. Subsequent iterations are used to deduce a final model which produces traveltimes residuals meeting predefined stopping criteria. Interpretation procedures of this type should include estimates of resolution, uncertainty, and uniqueness of the model parameters.

Reflection Method

The reflection method in near-surface seismology is an adaptation of theory and procedures pioneered in the petroleum exploration industry. A significant literature now exists describing near-surface reflection methodology, and numerous case studies are available illustrating diverse applications. Most practitioners of the reflection method use common midpoint (CMP) acquisition and data reduction because it takes full advantage of currently available digital recording systems and seismic processing algorithms. This section is therefore focused on an introductory explanation of the essential elements of the CMP reflection method. The discussion includes the hyperbolic signature of a reflection on a field record and CMP gather, the optimum window concept, normal moveout corrections, and horizon stacking. Sequential processing of a CMP gather is shown to illustrate the basic process leading to a stacked trace. An example of a fully processed reflection for the reflection traveltime from a horizontal interface is given by Hunter et al. (1984). Tutorials on various aspects of the CMP method are given by Knapp and Steeples (1986a, 1986b), Steeples and Miller (1990), and Baker (1999). Important updates from a special workshop held in 1996 are provided by Steeples et al. (1997) with further elaboration by Steeples and Miller (1998). Numerous recent case histories are given in the supplementary references of this chapter and provide the critical link between the tutorials and actual experience in a wide variety of geologic settings. Historical summaries of the near-surface reflection method are given by Hunter and Pullan (1989), Steeples et al. (1995), and Steeples (1998).

Hyperbola on a split-spread field record

A reflection is distinguished on a split-spread field record by the hyperbolic shape of its travel-time curve. Consider an idealized subsurface with a plane dipping interface separating two materials of contrasting seismic velocities $V_1$ and $V_2$, as shown in Figure 11. Simple geometric analysis of the reflected raypaths shows that the reflected-wave traveltime is a hyperbolic function of offset $x$:

$$T_{rw}(x) = T_0 \sqrt{1 + \frac{x^2 - 4hx \sin \xi}{4h^2}}, \quad (59)$$

where $\xi$ is the interface dip, $h$ is the depth measured normal to the interface from the source location at $x = 0$, and $T_0 = 2h/V_1$ is the traveltime for the normally incident raypath. If the derivative $dT_{rw}/dx$ is set equal to zero, the minimum traveltime is found to be $T_0 \cos \xi$, which occurs updip at $x = 2h \sin \xi$. In the case of a horizontal interface, the dip $\xi$ is zero and equation (59) reduces to a hyperbola that is symmetric in $x$ with minimum traveltime $T_0$ at $x = 0$:

$$T_{rw}(x) = T_0 \sqrt{1 + \frac{x^2}{4h^2}}. \quad (60)$$

Equation (60) is commonly rewritten by squaring both sides and simplifying terms to obtain the following expression for the reflection traveltime from a horizontal interface:

$$T_{rw}^2(x) = T_0^2 + \frac{x^2}{V_1^2}. \quad (61)$$

Although equations (59)–(61) are strictly applicable only to the idealized subsurface model shown in Figure 11, they are among the most robust results in seismology. Hyperbolic reflection signatures have been observed on countless field records acquired in a wide variety of geologic environments over a remarkable range of interface depths. An example showing a reflection hyperbola for a near-surface dipping interface is shown in Figure 12.

Optimum window

The range of offsets on a field record for which a near-surface reflection is observed with minimum interference from other seismic waves is known as the optimum window (Hunter et al., 1984). An idealized subsurface model with a plane horizontal interface separating two materials with contrasting seismic velocities provides a useful illustration.
of the optimum window concept (Figure 13). It is assumed that the velocity of the surficial layer is less than the velocity of the underlying layer \( V_1 < V_2 \), and that the large-amplitude surface-wave modes are limited to an approximately triangular region of relatively low apparent velocity. The optimum window for this simple case lies between the surface waves and the critical distance marking the emergence of the head wave.

It is possible to recover reflections from receivers inside the region of surface-wave interference by digital filtering. Steeples and Miller (1998) report that in their experience using the reflection method at over 100 sites, the dominant frequency of near-surface reflections — reflections with traveltimes less than 100 ms — is approximately double the dominant frequency of the surface waves. Thus, assuming sufficient dynamic range in the recording system, the interference represented by surface waves at near offsets can often be lessened by high-pass filtering. Expansion of the optimum window at large offsets is complicated by interference between the reflected waves and the direct and head waves; separation of these arrivals is problematic because their frequency content and apparent velocities are similar.

At offsets comparable to the reflector depth, Pullan and Hunter (1985) note that the reflection will undergo phase changes in response to reflection and transmission in the vicinity of the critical angle.

**Small offset hyperbola on a common midpoint gather**

Consider a configuration of source-receiver pairs symmetrically placed about a point on the surface known as the **common midpoint** (CMP); see Figure 14. The set of traces associated with source-receiver pairs \( S_1 - R_1, S_2 - R_2 \), and so on, when plotted against offset \( x \), is a **common midpoint gather or CMP gather**. Following Hubral and Krey (1980, 70), it is possible to develop a hyperbolic approximation for the traveltime \( T(x) \) of a reflected wave recorded on the traces of the CMP gather. It is this hyperbolic approximation that is exploited in the CMP reflection method.

Development of the hyperbolic approximation begins by writing \( T^2(x) \) as a Taylor series expansion about \( x = 0 \):

\[
T^2(x) = T^2(0) + b_1 x + b_2 x^2 + b_3 x^3 + \ldots, \tag{62}
\]

where the constants \( b_i \) are the Taylor series coefficients. The traveltime \( T(0) \) at zero offset is called the **normal incidence time**, because a reflected raypath starting and ending at a coincident source-receiver pair must be normally incident on the reflecting interface (Figure 15). Reciprocity requires \( T^2(x) \) to be an even function of \( x \). Thus, the odd-indexed coefficients \( b_1, b_3, \) and so on) must be zero, and the Taylor series [equation (62)] reduces to even powers of \( x \):

![Figure 11. Raypaths and traveltime curve for a reflection from a plane dipping interface (dip \( \xi \)) separating two materials of contrasting seismic velocities \( V_1 \) and \( V_2 \). The source (black square) and receivers (black circles) are located at the surface along the \( x \)-axis, with the source at the origin and receivers symmetrically located about the source. Distance \( h \) is depth measured normal to the interface from the source. Constant \( T_0 \) is the normally incident traveltime \( 2h/V_1 \). The reflection traveltime \( T_{\text{r}_1} \) is a hyperbola with minimum traveltime \( T_0 \cos \xi \) occurring at distance \( 2h \sin \xi \) updip from the source.](image-url)

![Figure 12. Reflection from a dipping interface recorded on a split spread. The interface is at a depth of about 35 m beneath the source, and dips approximately 25° with the down-dip direction to the left in the diagram (same geometry and orientation as Figure 11). The source is an 11.4-kg vertical slide hammer located at zero offset, and the receivers are geophones with 10-Hz natural frequency. Each trace has been band-pass filtered with a 15-30-350-500 Hz trapezoidal passband. (Courtesy L. M. Liberty.)](image-url)
It is important to keep in mind that in general, the remaining Taylor series coefficients ($b_2$, $b_4$, and so on) are complex functions of the subsurface velocity structure.

The properties of a Taylor series expansion of a continuous function ensure that the first two terms in equation (63) represent $T^2(x)$ to arbitrary accuracy for small enough offset. Therefore, if $x < \Delta x$, where $\Delta x$ is a sufficiently small positive number, terms beyond the $x^2$ term may be neglected, and the traveltime of a reflected wave observed on a CMP gather is given by the small-offset hyperbola (also called the small-spread hyperbola):

$$T^2(x) = T^2(0) + b_2 x^2 + \ldots$$  \hspace{1cm} (63)

For reasons that will become clear shortly, it is customary to denote the Taylor coefficient $b_2$ by $1/V^2_{NMO}$ so that the small-offset hyperbola [equation (64)] is written as follows:

$$T^2(x) = T^2(0) + \frac{x^2}{V^2_{NMO}} \quad x < \Delta x.$$  \hspace{1cm} (65)

where $V_{NMO}$ has dimensions of velocity and is called the normal moveout velocity. It turns out that the normal moveout velocity is given by simple exact expressions for several subsurface models involving homogeneous layers. In the following equations, $V_{NMO}$ is the normal moveout velocity for the reflection from the base of the indicated layer(s):

$$V_{NMO} = V_1 \quad \text{(single horizontal layer)},$$  \hspace{1cm} (66)

$$V_{NMO} = \frac{V_1}{\cos \xi} \quad \text{(single dipping layer)},$$  \hspace{1cm} (67)

and

$$V_{NMO} = \frac{1}{T(0)} \sum_{k=1}^{N} V_k \Delta h_k \quad (N \text{ horizontal layers}),$$  \hspace{1cm} (68)

where $V_1$ is the velocity of the single layer, $\xi$ is the dip of the single dipping layer, and $V_k$ and $\Delta h_k$ are the velocity and thickness of layer $k$, respectively, of $N$ horizontal layers; see Figure 16. Formulas for $V_{NMO}$ for more complicated subsurface models that include the results in equations (66)–(68) as special cases are developed by Hubral and Krey (1980).
Normal moveout

The normal moveout or NMO of a reflection recorded on a CMP gather at offset \( x \) is defined to be the difference between the reflection traveltime \( T(x) \) and the normal incidence time \( T(0) \) for that reflection:

\[
\Delta_{\text{NMO}}(x) = T(x) - T(0).
\] (69)

Substituting \( T(x) \) from the small-offset hyperbola (65) into equation (69) gives the NMO under the small offset assumption:

\[
\Delta_{\text{NMO}}(x) = \frac{x^2}{V_{\text{NMO}}^2} - T(0) \quad x < \Delta x. \tag{70}
\]

As can be seen from equations (65) and (70), the normal moveout velocity is the key parameter at small offsets in determining the shape of the reflection hyperbola and the size of the normal moveout. In other words, as the normal moveout velocity \( V_{\text{NMO}} \) increases without limit, the normal moveout \( \Delta_{\text{NMO}} \) decreases toward zero, and the small-offset hyperbola flattens to a horizontal straight line given by \( T(x) = T(0) \).

Stacking hyperbola and velocity analysis

It has been shown above that under the small offset assumption, the traveltime \( T(x) \) of a reflection observed on a CMP gather can be represented by the small-offset hyperbola defined by equation (65). However, as offset increases, the agreement between \( T(x) \) and the small-offset hyperbola deteriorates in a manner that depends on the subsurface structure. The discussion below describes an empirical procedure used to obtain a better overall hyperbolic approximation to \( T(x) \) over the full range of CMP offsets. Unlike the small-offset hyperbola, this new hyperbolic approximation is not guaranteed to match \( T(x) \) over any offset range, but it does lead to the very useful procedure known as horizontal stacking that is almost universally employed in reflection data processing.

Suppose the reflection traveltime \( T(x) \) is measured on \( M \) traces of a CMP gather to give traveltime data \([x_i, T(x_j)]\), where subscript \( j = 1, 2, \ldots, M \) indicates the trace number. These traveltime data may be used to constrain the parameters of the stacking hyperbola which is defined as follows:

\[
T_{\text{ST}}^2(x) = T_{\text{ST}}^2(0) + \frac{x^2}{V_{\text{ST}}^2},
\] (71)

where \( T_{\text{ST}}(0) \) and \( V_{\text{ST}} \) are adjustable stacking parameters, and \( V_{\text{ST}}^2 \) is known as the stacking velocity. Extension of a hyperbolic approximation for traveltime \( T(x) \) to all offsets of the CMP gather (not just small offsets) can be accomplished by finding the values of the stacking parameters \( T_{\text{ST}}(0) \) and \( V_{\text{ST}} \) that provide some best fit to the \([x_j, T(x_j)]\) traveltime data. In general, the stacking parameters \( T_{\text{ST}}(0) \) and \( V_{\text{ST}} \) will not equal \( T(0) \) and \( V_{\text{NMO}} \) of the small-offset hyperbola, although they are often considered to be approximations to them.

Perhaps the simplest way to determine the stacking parameters is to plot \( T^2(x) \) as a function of \( x^2 \) and then perform a linear least-squares fit to obtain \( T_{\text{ST}}^2(0) \) as the intercept and \( 1/V_{\text{ST}}^2 \) as the slope. However, this procedure is just one method of velocity analysis, a term used to describe a wide variety of processes developed over the years for determining the stacking parameters for each reflection on a CMP gather; see Yilmaz (2001) for a thorough discussion and summary. The key point underlying velocity analysis is that a properly determined stacking hyperbola closely follows a reflection on the CMP gather and thereby guides constructive trace-by-trace summing of the reflection signal. Regardless of the details involved in velocity analysis, the final result is the definition of a stacking velocity, and hence a stacking hyperbola, for each time increment on the zero-offset axis of a CMP gather.

Flattening a reflection by the normal moveout correction

Once the stacking velocities are determined for a CMP gather, the reflections on that gather may be flattened by a process known as the normal moveout correction. For a
given time $T_j(0)$ on the zero offset axis, equation (71) is used to compute $T_{ST}(x_i)$, where $x_i$ is the offset of trace $i$ of the CMP gather. $T_{ST}(0)$ is given by $T_j(0)$, and the stacking velocity $V_{ST}$ is the stacking velocity associated with $T_j(0)$:

$$T_{ST}(x_i) = \sqrt{T_j^2(0) + \frac{x_i^2}{V_{ST}^2}}. \quad (72)$$

The trace value at offset $x_i$ and time $T_{ST}(x_i)$ is then mapped to offset $x_i$ and time $T_j(0)$. By repeating this operation for all traces, a reflection with a hyperbolic traveltime curve approximated by equation (72) is aligned or flattened at a time that closely corresponds to its normal incidence time. Because coherent events that are not aligned with a stacking hyperbola are not flattened, the normal moveout correction is seen as a mechanism for separating reflections from all other events, including multiple reflections. Figure 17 provides an example of the result of normal moveout corrections applied to a CMP gather acquired in Oregon.

Application of the normal moveout correction to reflection waveforms of finite duration results in a waveform distortion known as NMO stretch (Barnes, 1992). The effect is most serious at small normal incidence times and large offsets and therefore has considerable significance for near-surface CMP data. The problem can be handled by careful selection of allowable stretch parameters in a processing algorithm that mutes unacceptably stretched reflections (Miller, 1992).

**CMP reflection method**

The preceding development of the hyperbolic representation of reflection traveltime curves and the normal moveout correction is a sufficient basis for summarizing the CMP reflection method. Although vast in its details, the CMP method may be viewed as consisting of the five essential steps listed below.

1) Acquisition of CMP gathers is carried out for a sequence of common midpoints along a profile on the earth’s surface. A highly efficient procedure known as roll-along acquisition is used to acquire the necessary data.

2) Stacking hyperbolas for each CMP gather are determined using velocity analysis.

3) Each reflection on each CMP gather is flattened by application of the normal moveout correction.

4) The flattened traces of each CMP gather are summed to achieve one stacked trace for each gather. This process is known as horizontal stacking and has the effect of enhancing reflections while canceling those events that do not align properly as a result of the normal moveout corrections. The number of traces summed to get a stacked trace is known as the fold; the fold is typically 6–30 in near-surface applications.

5) The stacked traces are plotted side-by-side in CMP order to obtain a seismic section of the subsurface. Alignments of reflections on the seismic section mark continuous geologic interfaces with significant reflection coefficients; interruptions of these alignments may indicate faults.

**Figure 17.** Sequential processing of a common midpoint (CMP) gather. (a) Original CMP gather. (b) After application of a trapezoidal band-pass filter (40-80-500-800 Hz). (c) After NMO corrections with stretch mute. (d) After top and bottom mutes. In each panel, traces are shown with trace gain applied (i.e., each trace is normalized to its maximum amplitude). (Courtesy L. M. Liberty.)
An example of a seismic section acquired as part of a seismic hazards study in Oregon is given in Figure 18. The reflection alignments are clearly visible in the uninterpreted section in the upper part of the figure; a geologic interpretation is superimposed on the section in the lower part.

Static corrections

The hyperbolic shape of a reflection traveltime curve on a field record or CMP gather may be distorted by variation in the elevation of sources and receivers, and by lateral changes in velocity in the very shallow subsurface (especially above the water table). These distortions correspond to higher order terms in the Taylor series expansion (63) and are therefore not represented by the small-offset hyperbola. Because the distortions are also not represented by the stacking hyperbola, reflections are not adequately flattened prior to horizontal stacking. The result can be a serious degradation of the quality of the seismic section. Static corrections are time shifts used to adjust the traces in a field record or CMP gather toward an ideal condition where the terrain is flat and without shallow lateral velocity variations.

Whole-trace shifts in time that represent vertical translation of the source and receiver to a common elevation or datum are called elevation static corrections or datum static corrections. The accuracy of elevation static corrections is critically dependent on choosing the correct average velocity to represent the material between the surface and the datum. Refraction static corrections are used to correct for lateral variations in the thickness and velocity of the surficial low-velocity layer. In the simplest and perhaps most reliable situation in which the low-velocity layer is the unsaturated zone above the water table, refraction static corrections are determined from deviations in the arrival time of the water-table head wave (Hunter et al., 1984). Residual static corrections are used to correct short wavelength variations in shallow velocity beneath individual sources and receivers. Regardless of the type of static correction, Steeples and Miller (1998) caution that static corrections on near-surface seismic reflection data require considerable care because the time shifts are comparable to the dominant periods of the reflections.

Borehole ties to lithology

The interpretation of reflection data in terms of geologic interfaces requires development of a correspondence between normal incidence time on the seismic section and depth in a borehole with lithologic control. A comprehensive discussion of the experimental aspects of this problem is provided by Hunter et al. (1998), who describe downhole seismic logging techniques for both P-waves and S-waves.

Other considerations

Numerous pitfalls exist in the acquisition and processing of seismic reflection data in the near-surface environment. These pitfalls are summarized concisely by Steeples and Miller (1998) in an article describing experience gained during 25 years by the authors and their colleagues at numerous field sites representing many geologic environments. Perhaps the single most important action that can be taken to avoid problems is to confirm interpreted reflections on the seismic section by comparison with the minimally processed field records or CMP gathers. If an interpreted reflection cannot be confirmed in this way, then there is the possibility that it has been created as an artifact of acquisition and/or processing.

The treatise by Yilmaz (2001) is a very complete and readable exposition of the most important results from the massive literature on seismic data processing. Although the context of Yilmaz (2001) is the reflection method as practiced by the petroleum industry, it nonetheless serves as a valuable reference for processing reflection data acquired for near-surface investigations. Adaptation of the processing methods from the petroleum exploration industry to the near surface requires careful examination of the underlying assumptions.

For example, Steeples and Miller (1998) report that deconvolution does not work properly on most near-sur-
face reflection data, because the reflection wavelet varies both laterally and with depth, and because the series of reflection coefficients is not modeled well as a random time series. As another example, Black et al. (1994) find that migration of near-surface seismic sections is often not required for low subsurface velocities and small reflector depths relative to spread lengths. Migration is the process of repositioning dipping reflectors properly in time and space on a seismic section; examples of successful migrations are demonstrated by Bradford et al. (1998) and Pasasa et al. (1998). Migration has been carried out for the interpreted seismic section in the lower part of Figure 18.

Recent developments

Sustained effort in the acquisition and processing of 3D near-surface reflection data is probably the most notable of the recent developments; see the examples by Buker et al. (1998, 2000), Siahkoohi and West (1998), Spitzer et al. (2001), Spitzer et al. (2003), and Bachrach and Mukerji (2004a, b). Coupled with previously mentioned attempts to speed the deployment of receivers (see “Seismic Sources and Field Instrumentation”), this work is significant because the application of 3D reflection methods is often stymied by high costs. More difficult processing problems are also being addressed, such as those associated with very large subsurface velocity gradients that can occur, for example, in the change from unsaturated to saturated conditions near the water table (see Miller and Xia, 1998 and Bradford, 2002a, b). Another important development is the combined use of both ground-penetrating radar and very high-resolution seismic reflection methods on the same imaging problem; for example, see Cardimona et al. (1998), Baker et al. (2001), and Liberty et al. (2003). Similarly, Ghose and Goudswaard (2004) examine the process of integrating S-wave reflection data with cone penetration tests.

Some investigators are working on the relationships between seismic reflection measurements and material properties, e.g., Bachrach and Mukerji (2004c). Finally, there has been interest in defining a practical minimum reflector depth. Recent experiments have shown that reflections from a depth of approximately 1 m may be recorded with a dominant frequency of about 600 Hz using surface sources and receivers (Steeples, 1998).

Surface-Wave Method

The surface-wave method was developed in response to needs in geotechnical engineering and S-wave reflection seismology for a noninvasive technique for estimating the in situ S-wave velocity of near-surface materials. Conventional implementation of the method involves the recording of Rayleigh waves on vertical-component receivers, using the Rayleigh-wave data to estimate the phase-velocity dispersion curve, and then applying the methods of geophysical inversion to the dispersion curve to obtain the S-wave velocity as a function of depth.

The SASW method (spectral analysis of surface waves) and the MASW method (multichannel analysis of surface waves) are the two primary techniques that have emerged for field data acquisition and subsequent estimation of the dispersion curve. Surface waves of broad bandwidth are desirable to ensure that the recorded dispersion is representative of the full depth range of interest. Maximum depths depend on the application and vary from a few decimeters in pavement systems to a several decameters in site investigations.

Most of the inversion schemes in current use adopt a horizontally layered earth model in which each layer is a homogeneous and isotropic, linearly elastic medium. These inversion schemes differ primarily in the computation of the forward model and the method of optimization. Applications of near-surface S-wave velocity information include foundation dynamics, pavement analysis, soil improvement, liquefaction potential, and static corrections for S-wave reflection data.

Spectral analysis of surface waves method

The spectral analysis of surface waves method (SASW) is distinguished by the use of cross-spectral techniques to determine a phase-velocity dispersion curve for Rayleigh waves. A typical field experiment uses a pair of identical vertical-component receivers \( G_1 \) and \( G_2 \) of low natural frequency (e.g., \( f_0 = 1 \) Hz) placed symmetrically about a fixed point on the surface (Figure 19). A source suitable for the generation of Rayleigh waves is placed along the line established by the two receivers. The source is activated at both forward and reverse source points with offset to the nearest receiver equal to the receiver separation. The seismic signals from \( G_1 \) and \( G_2 \) for each source activation are recorded by a dual-channel digital recorder capable of spectral analysis in the field; storage of each trace pair for future processing is usually on a portable computer. Data are acquired for several different receiver separations where each separation is generally twice the previous separation. The corresponding offset increase usually requires an increase in source energy to provide sufficiently strong surface waves at the receivers.

The reduction of SASW data is based on spectral analysis of each trace pair. Stochastic estimates of the cross spectrum and coherence function are of particular importance. However, to understand the use of the cross spectrum in the SASW method, it is possible to disregard the stochastic nature of the recorded signals and consider a de-
terministic (nonrandom) pair of continuous traces $G_1(t)$ and $G_2(t)$ that record the Rayleigh wave from a single source activation. The crosscorrelation function $G_{12}(t)$ for $G_1(t)$ and $G_2(t)$ is defined as follows (see Appendix A):

$$G_{12}(t) = \int G_1(\tau)G_2^*(t+\tau)\,d\tau$$

(73)

The cross spectrum $\tilde{G}_{12}(\omega)$ is the Fourier integral transform of $G_{12}(t)$, and may be expressed in polar form as follows:

$$\tilde{G}_{12}(\omega) = |\tilde{G}_{12}(\omega)|e^{i\phi_{12}(\omega)}$$

(74)

where $\phi_{12}(\omega)$ denotes the phase spectrum of the cross spectrum. The cross spectrum may also be written in terms of the individual Fourier integral transforms $\tilde{G}_1(\omega)$ and $\tilde{G}_2(\omega)$ of traces $G_1(t)$ and $G_2(t)$, respectively, by using the crosscorrelation theorem (see Appendix A):

$$\tilde{G}_{12}(\omega) = \tilde{G}_1(\omega)\tilde{G}_2^*(\omega)$$

(75)

where the asterisk indicates the complex conjugate. The significance of equation (75) is clarified by writing $\tilde{G}_1(\omega)$ and $\tilde{G}_2(\omega)$ in polar form with phase spectra $\phi_1(\omega)$ and $\phi_2(\omega)$, respectively:

$$\tilde{G}_1(\omega) = \tilde{|G}_1(\omega)|e^{i\phi_1(\omega)}$$

and

$$\tilde{G}_2(\omega) = \tilde{|G}_2(\omega)|e^{i\phi_2(\omega)}$$

(76)

Substitution of the polar forms (76) into the crosscorrelation theorem (75) gives

$$\tilde{G}_{12}(\omega) = \tilde{|G}_1(\omega)|\tilde{|G}_2(\omega)|e^{i(\phi_2(\omega)-\phi_1(\omega))}$$

(77)

Comparison of equations (74) and (77) shows that the phase spectrum $\phi_{12}(\omega)$ of the cross spectrum gives the phase difference of trace $G_2(t)$ relative to trace $G_1(t)$:

$$\phi_{12}(\omega) = \phi_2(\omega) - \phi_1(\omega)$$

(78)

As is discussed next, the result in equation (78) permits $\phi_{12}(\omega)$ to be used to compute the phase-velocity dispersion curve $c(\omega)$ of the Rayleigh waves recorded by receivers $G_1$ and $G_2$.

The vertical component $u_z$ of a single mode of the Rayleigh wave displacement at receivers $G_1$ and $G_2$ can be written for frequency $\omega$ as follows:

$$G_2: \quad u_z(x, z = 0, t) = w \cos\left[\omega t - \kappa_0(\omega)x\right]$$

$$G_1: \quad u_z(x + d, z = 0, t) = w \cos\left[\omega t - \kappa_0(\omega)(x + d)\right]$$

(79)

where $G_1$ and $G_2$ are located at horizontal positions $x + d$ and $x$, respectively, the fundamental mode is assumed with wavenumber $\kappa_0(\omega)$, constant $w$ is the fundamental mode amplitude for frequency $\omega$, and both receivers are on the surface $z = 0$. It is important to realize that the fundamental mode is used here only as an example. Any mode can be used in the discussion, but there is an implicit assumption that a single mode dominates the displacement at the receivers.

It is evident from the cosine functions in equation (79) that the phase difference $\Delta \phi(\omega)$ of the displacement at receiver $G_2$ relative to the displacement at receiver $G_1$ is the product of the geophone separation $d$ and wavenumber $\kappa_0(\omega)$:

$$\Delta \phi(\omega) = d\kappa_0(\omega).$$

(80)

Dividing both sides of equation (80) by frequency $\omega$ and rearranging gives the Rayleigh-wave phase velocity $c(\omega)$:

$$c(\omega) = \frac{\omega}{\kappa_0(\omega)} = \frac{\omega d}{\Delta \phi(\omega)}.$$  

(81)

The displacements in equation (79) are converted into the traces $G_1(t)$ and $G_2(t)$ by convolution with the impulse response of the field recording system. Because the phase changes introduced by the recording system can be assumed to cancel in the subtraction in equation (78), the

Figure 19. Typical data acquisition scheme for SASW method. Three individual spreads are shown with each spread centered on the same point $P$ on the surface. Each spread is a reversed inline offset spread. Black squares are sources and black circles are receivers (identified as $G_1$ and $G_2$ in text). For each spread, the offset between each source and the nearest receiver equals the receiver separation (i.e., the near offset $\Delta x$ equals the receiver separation $d$ as shown for the third spread). The spacing between the sources and receivers for a given spread is twice that of the preceding spread.
phase spectrum \( \phi_{12}(\omega) \) is equivalent to \( \Delta \phi(\omega) \). Thus, estimation of the phase-velocity dispersion curve can be reduced to determining \( \phi_{12}(\omega) \):

\[
c(\omega) = \frac{\omega}{\kappa_0(\omega)} = \frac{\omega d}{\phi_{12}(\omega)}. \tag{82}
\]

Although presentation of the dispersion curve as a function of frequency is common in seismology, the conversion of \( c(\omega) \) to a function of wavelength \( c(\lambda) \) is often used in geotechnical engineering. The conversion is accomplished by recalling that the wavelength \( \lambda \) for a given frequency \( \omega \) is given by \( \lambda = 2\pi c(\omega)/\omega \).

Estimation of the dispersion curve in the SASW method is subject to limitations on the longest usable wavelength. Because of near-field effects, Stokoe et al. (1994) recommend that any wavelength \( \lambda \) for which a phase-velocity estimate \( c(\lambda) \) is obtained should satisfy the following criterion:

\[
\lambda < 2\Delta x, \tag{83}
\]

where \( \Delta x \) is the offset between source and nearest receiver (recall that \( \Delta x = d \) in the normal SASW field protocol where \( d \) is the distance between receivers). For pavement systems, Sanchez-Salinero et al. (1987) suggest a much stricter criterion:

\[
\lambda < \frac{\Delta x}{2}. \tag{84}
\]

As a consequence of these restrictions on wavelength, the dispersion curve is composed of overlapping segments with greater offsets used for measuring phase velocities for longer wavelengths. A smoothing procedure is used to convert the overlapping segments to a single composite dispersion curve.

It is well known that surface wave propagation involves multiple modes and each mode is generally associated with a different dispersion curve (Tokimatsu et al., 1992). Thus, an assumption implicit in the procedure outlined above is that Rayleigh waves of a single mode dominate the seismograms at all wavelengths so that this dominant mode is the primary determinant of the dispersion curve. Analysis of wave propagation in an elastic half-space with shear modulus increasing with depth was carried out by Vrettos (1991). One finding of the Vrettos (1991) study is that the phase velocity of the fundamental mode Rayleigh wave approximates the phase velocity of the total half-space response beyond a short source-receiver distance (a few wavelengths). However, Gucunski and Woods (1991a,b) stress that higher mode Rayleigh waves can dominate surface wave propagation for subsurface profiles with a low-velocity layer between higher velocity layers.

### Use of the coherence function in the SASW method

The **coherence function** \( \gamma^2(\omega) \) is used as a measure of data quality in the SASW method and is defined as follows (Jenkins and Watts, 1968, 352):

\[
\gamma^2(\omega) = \frac{\mathcal{G}_{12}(\omega)^2}{\mathcal{G}_{11}(\omega)\mathcal{G}_{22}(\omega)}, \tag{85}
\]

where the traces \( G_1(t) \) and \( G_2(t) \) are now viewed as stochastic processes. The functions \( \mathcal{G}_{11}(\omega) \) and \( \mathcal{G}_{22}(\omega) \) are the power spectra of traces \( G_1(t) \) and \( G_2(t) \), respectively, and \( \mathcal{G}_{12}(\omega) \) is the cross spectrum.

The use of the coherence function as a measure of data quality can be explained by supposing that \( G_2(t) \) is the convolution of \( G_1(t) \) with some impulse response \( h(t) \) (such as a simple delay) plus independent noise \( N(t) \):

\[
G_2(t) = h(t) * G_1(t) + N(t). \tag{86}
\]

The ratio of the power spectrum of the signal at receiver \( G_2 \) to the power spectrum of the noise for this model can be expressed in terms of the coherence function (Jenkins and Watts, 1968, 351–352):

\[
\frac{\mathcal{G}_{22}(\omega)}{\mathcal{N}_{NN}(\omega)} = \frac{1}{1 - \gamma^2(\omega)}. \tag{87}
\]

where \( \mathcal{N}_{NN}(\omega) \) is the power spectrum of the noise. Thus, at a particular frequency \( \omega \), if the signal at \( G_2 \) is all noise, then the ratio in equation (87) is unity and the coherence function is zero; but if the noise is zero at \( G_2 \), then the ratio is infinite and the coherence function is unity.

As a simple example, suppose \( G_1(t) = s_c G(t) \), where \( s_c \) is a positive real number and \( G(t) \) is a stochastic process that constitutes the essential aspects of the surface-wave signal, and further suppose that \( G_2(t) \) is a delayed version of \( G_1(t) \) plus independent noise:

\[
G_2(t) = s_c G(t - \tau) + N(t). \tag{88}
\]

where \( G(t) \) and \( N(t) \) have zero mean and equal variance. Scaling factor \( s_c \) is introduced as a way of adjusting the strength of the signal relative to the noise. The coherence function in this case can be shown to be independent of frequency and totally dependent on the scaling (Jenkins and Watts, 1968, 357):

\[
\gamma^2 = \frac{s_c^2}{1 + s_c^2}. \tag{89}
\]

Thus, if the signal is small relative to the noise (\( s_c \) approaches zero), then the coherence function also approaches zero. On the other hand, if the signal is large rel-
ative to the noise, (τ, approaches infinity), then the coherence function approaches unity.

Phase velocities at frequencies with $\gamma^2(\omega)$ significantly less than one are disregarded in the SASW method. An example of the typical use of the coherence function in the SASW method is given by Nazarian and Desai (1993).

**Recent developments in the SASW method**

Notable advances in the traditional application of the SASW method include automation of dispersion curve determination (Nazarian and Desai, 1993), and automation of the inversion process (Yuan and Nazarian, 1993), including the incorporation of multiple modes in the inversion (Ganji et al., 1998). Underwater applications of the SASW method are discussed by Luke and Stokoe (1998). The potential of the SASW method to detect a subsurface obstacle such as a cavity is investigated numerically by Ganji et al. (1997). Rix et al. (2000) and Lai et al. (2002) have extended the SASW method to the determination of the damping ratio.

**Multichannel analysis of surface waves method**

The multichannel analysis of surface waves method (MASW) is distinguished from the SASW method by the analysis of field records acquired with considerably more receivers than the pair of receivers used for SASW acquisition. According to Park et al. (1999), the primary advantages of using multiple channels are the recognition and isolation of noise (defined as all wave types other than fundamental mode Rayleigh waves) and the speed and redundancy of data acquisition. In addition, multichannel data processing can be used to separate the dispersion curves for the fundamental and higher modes. The dispersion curves for higher modes can be used to help constrain the inversion for the shear-wave velocity-depth profile.

A typical field experiment uses a vertical impulsive or swept frequency source to generate Rayleigh waves that are recorded by an inline offset spread of identical receivers of relatively low natural frequency (1 < $f_0$ < 10 Hz). As in the SASW method, it is important that wavelengths should be smaller than twice the smallest inline offset to avoid near-field effects and to promote dominance of the fundamental mode in the recorded data. The far offset is limited by the distance at which higher frequency surface waves are significantly contaminated by reflected body waves. In the ideal case in which a single Rayleigh mode dominates the noise and all other modes, a multichannel swept frequency record (acquired with a swept frequency source or transformed from an impulsive record) will show a distinctive trace-to-trace linear waveform coherency (Park et al., 1999). Any breakdown in this coherency serves as a guide to modifications in the acquisition and subsequent data-processing protocols. Comparison of results from the MASW method with those of downhole measurements have been carried out by Xia et al. (2000).

Several forms of multichannel analysis can be used to deduce the phase-velocity dispersion curves. Park et al. (1999) point out that it is possible to simply measure the linear slope of each frequency component on the field records. McMechan and Yedlin (1981) show that an image of a dispersion curve is obtained directly by a slant stack (the $p-\tau$ domain) followed by a one-dimensional Fourier transform over $\tau$ (the $p-\omega$ domain), where $\tau$ is the time intercept, $p$ is the horizontal slowness, and $\omega$ is the frequency. The loci of maximum amplitudes in the $p-\omega$ domain define the phase-velocity dispersion curves for different modes. Gabriels et al. (1987) used $f-k$ (frequency-wavenumber) transformation to achieve separation of the fundamental and higher mode Rayleigh waves. As in the case of the $p-\omega$ technique, the loci of maximum amplitudes in the $f-k$ domain define the phase-velocity dispersion curves for the different modes; the corresponding phase velocity for any point on a locus is given by the $f/k$ ratio.

**Inversion: General concepts**

The inversion of geophysical data typically begins with a *discrete parameterization* of the subsurface in terms of a finite-length vector $\mathbf{p}$ of discrete parameters. For a given discrete parameterization, an infinite number of different realizations of $\mathbf{p}$ exists, one realization for each set of physically reasonable numerical values that can be assigned to the elements of $\mathbf{p}$. The complete set of possible realizations of $\mathbf{p}$ is termed parameter space. The primary purpose of inversion is to select a specific parameter vector $\mathbf{p}_c$ from parameter space as representative of the actual properties of the subsurface.

A necessary tool for the selection of $\mathbf{p}_c$ is some measure of consistency (known as the *objective function*) between the observed geophysical data and a set of corresponding theoretical predictions called the theoretical data; the theoretical data are computed from a trial parameter vector $\mathbf{p}_{\text{trial}}$ and a satisfactory theory of the relevant phenomenon called the forward model or forward problem. The objective function for a succession of trial parameter vectors is evaluated as a means of converging on a satisfactory choice for $\mathbf{p}_c$.

Additional criteria are often used to reduce nonuniqueness in the selection of $\mathbf{p}_c$, including the minimization (or maximization) of certain global characteristics such as the magnitude or spatial variation of the parameter vector; these additional criteria are often incorporated into the objective function.

Given this general description of geophysical inversion, the basic components of an inversion procedure for surface-wave data may be listed as follows:
A finite number of points on an observed dispersion curve

A discrete parameterization of the subsurface, typically as a horizontally layered medium

A solution to the problem of computing theoretical dispersion data for a given layered earth model (i.e., the solution to the forward problem)

An objective function that decreases with increasingly desirable characteristics of the layered earth model

A search process that considers a succession of trial layered earth models and selects a model that is optimum in the sense of minimizing the objective function

A process to appraise the uniqueness, resolution, and variance of the optimum layer parameters

General references for geophysical inversion theory include the compilation edited by Lines (1988), and the textbooks by Tarantola (1987), Menke (1989), Parker (1994), and Scales and Smith (1994).

Inversion of surface wave dispersion curves

A stack of \( L \) horizontal layers over a half space (Figure 20) is the most common discrete parameterization for inversion of surface wave dispersion curves. Each layer is assumed to be a homogenous and isotropic linearly elastic medium. The parameters for layer \( j \) are typically chosen to be the thickness \( h_j \), S-wave velocity \( V_{Sj} \), P-wave velocity \( V_{Pj} \), and density \( \rho_j \). Equivalent parameters in each layer can be defined based on the general relationships between the P-wave and S-wave velocities, the elastic constants, and the density (e.g., an equivalent set of layer parameters is thickness \( h_j \), Lamé modulus \( \lambda_j \), shear modulus \( \mu_j \), and density \( \rho_j \)). If Love waves are under consideration, then it is unnecessary to include the P-wave velocity or the Lamé modulus. A parameter vector \( \mathbf{p} \) with \( M = 4(L+1) \) parameters for Rayleigh waves, or \( M = 3(L+1) \) parameters for Love waves, is used to represent the complete set of all parameters.

The observed data consist of \( N > M \) phase velocities \( c(\omega_i) \) measured for a particular surface-wave type (Rayleigh or Love) and mode, and at a given set of frequencies \( \omega_i \):

\[
\mathbf{c} = \left[ c(\omega_1), c(\omega_2), \ldots, c(\omega_N) \right] . \quad (90)
\]

These data are to be compared with \( N \) theoretical phase velocities \( c_t(\omega_i) \) computed for the same surface-wave type, mode, and frequencies \( \omega_i \):

\[
\mathbf{c}_t = \left[ c_t(\omega_1, \mathbf{p}), c_t(\omega_2, \mathbf{p}), \ldots, c_t(\omega_N, \mathbf{p}) \right] , \quad (91)
\]

where trial values are used for the elements of the parameter vector \( \mathbf{p} \).

Computation of theoretical phase velocities for a layered linearly elastic subsurface is the forward problem in this case. It is typically a process of finding the roots of a dispersion equation or period equation which represents the constraints imposed on surface wave propagation in the layered model:

\[
f(\kappa, \omega, \mathbf{p}) = 0. \quad (92)
\]

Given a frequency \( \omega \) and parameter vector \( \mathbf{p} \), the dispersion equation defines the wavenumbers that can exist. These wavenumbers are denoted here by \( \kappa_n(\omega, \mathbf{p}) \) for integer \( n \), and are ordered so that \( \kappa_1(\omega, \mathbf{p}) > \kappa_2(\omega, \mathbf{p}) > \ldots > \kappa_2(\omega, \mathbf{p}) \) and so on. The fundamental mode has wavenumber \( \kappa_1(\omega, \mathbf{p}) \), and the higher modes are associated with the successively smaller wavenumbers. The theoretical phase velocity at frequency \( \omega \) for mode \( n \) is computed by the ratio of frequency to wavenumber.
Thus, by finding the roots of the dispersion equation (92) and computing the phase velocity using equation (93), a phase-velocity dispersion curve for each mode can be constructed.

Equations (92) and (93) represent an overview of the forward problem for a surface-wave inversion. Numerous specific methods exist for computing theoretical dispersion curves for surface waves for a layered elastic half space. These include the Thomson-Haskell matrix method (Thomson, 1950; Haskell, 1953), and the approaches credited to Knopoff and colleagues (Knopoff, 1964; Schwab and Knopoff, 1970, 1972). The Thomson-Haskell matrix method is a special case of the more general formulation of elastic wave propagation in a vertically heterogeneous medium introduced by Gilbert and Backus (1966) and known as the propagator matrix method. Other methods include the method of reflection and transmission coefficients (Kennett, 1983), the stiffness matrix method (Kausel and Roesset, 1981), and the numerical integration method (Takeuchi and Saito, 1972).

Given a discrete parameterization and the solution to the forward problem, the inversion is carried out by searching parameter space for one parameter vector \( \mathbf{p}_a \) that minimizes an objective function. The most basic objective function \( \Phi(\mathbf{p}) \) is the cumulative squared discrepancy between the observed and theoretical data:

\[
\Phi(\mathbf{p}) = (\mathbf{c} - \mathbf{c}_\mathbf{p})^T (\mathbf{c} - \mathbf{c}_\mathbf{p}),
\]

where the superscript \( T \) indicates transpose. The search for \( \mathbf{p}_a \) may proceed iteratively through a limited portion of parameter space from an initial guess \( \mathbf{p}_0 \) (e.g., by means of solving a sequence of local linear discrete inverse problems), or it may be a global search which is conducted over the entire parameter space.

Regardless of the search strategy, the inversion of surface-wave data generally suffers from nonuniqueness in the sense that more than one parameter vector can be regarded as acceptably minimizing the objective function. Nonuniqueness may be addressed by adding a priori information (e.g., it is common to fix the values of the P-wave velocity and density in an inversion of Rayleigh wave data because the inversion is relatively less sensitive to these parameters), and by adding a reasonable global constraint (e.g., requiring the S-wave velocities in \( \mathbf{p}_0 \) to be within a certain range). Questions of uniqueness are tied closely to the concepts of parameter resolution (the degree to which small features can be distinguished) and parameter variance (expected statistical variability). An understanding of parameter resolution and variance is obtained by reference to one of the previously cited general sources.

The literature describing the inversion of surface wave data is extensive if consideration is given to all scales of application. Key concepts for discrete linear inversion are identified and thoroughly discussed by Wiggins (1972); this work has influenced many later investigations. Recent papers that focus on various aspects of the inversion of surface-wave data for near-surface properties include Rix and Leipski (1991), Addo and Robertson (1992), Yuan and Nazarian (1993), Ganji et al. (1998), and Xia et al. (1999).

Inversion based on neural network theory is developed by Michaels and Smith (1997) and by Gucunski et al. (1998). The neural network papers and the work by Ganji et al. (1998) address the problem of including multiple modes.

A recent contribution by Forbriger (2003a, b) uses a Fourier-Bessel expansion to implement full-wavefield inversion; this method accounts for higher modes and leaky modes without identification of dispersion curves. The assumption of a horizontally layered subsurface model to account for dispersion has been called into question by van Wijk and Levshin (2004), who show that near-surface heterogeneity can give rise to dispersion without layering. Additional papers on surface-wave inversion are listed in the selected references at the end of this chapter.

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Appendix A

Cross correlation and autocorrelation

The cross correlation of two real functions $h_1(t)$ and $h_2(t)$ gives the cross correlation function $h_{12}(t)$ defined as follows:

$$h_{12}(t) = h_1(t) \ast h_2(t) = \int_{-\infty}^{\infty} h_1(\tau)h_2(t+\tau) \, d\tau.$$  \hspace{1cm} (A-1)

where the symbol $\ast$ indicates the operation of convolution. The cross correlation function is a measure of the similarity of the two functions at different lags or shifts represented by the variable $t$. The cross correlation of two different functions is not commutative, but the following relationship holds:

$$h_{12}(t) = h_{21}(-t).$$  \hspace{1cm} (A-2)

A change of variables demonstrates that the cross correlation of functions $h_1(t)$ and $h_2(t)$ can be computed by convolution:

$$h_1(-t) \ast h_2(t) = h_1(t) \ast h_2(t) = h_{12}(t).$$  \hspace{1cm} (A-3)

The autocorrelation of a real function $h(t)$ is the cross correlation of $h(t)$ with itself and results in an even function.

Cross spectrum and crosscorrelation theorem

The cross spectrum $\tilde{h}_{12}(\omega)$ is the Fourier integral transform of the cross correlation function $h_{12}(\omega)$:

$$\tilde{h}_{12}(\omega) = \int_{-\infty}^{\infty} h_{12}(t)e^{-i\omega t} \, dt.$$  \hspace{1cm} (A-4)

Given the following Fourier integral transform pairs:

$$h_1(t) \leftrightarrow \tilde{h}_1(\omega)$$
$$h_2(t) \leftrightarrow \tilde{h}_2(\omega)$$
$$h_{12}(t) \leftrightarrow \tilde{h}_{12}(\omega)$$  \hspace{1cm} (A-5)

the crosscorrelation theorem may be stated as follows:

$$\tilde{h}_{12}(\omega) = \tilde{h}_1(\omega)\tilde{h}_2(\omega)$$  \hspace{1cm} (A-6)

where the asterisk indicates the complex conjugate.

Appendix B

This appendix is devoted to a detailed development of the geophone equation which is given in the text as equation (10). Although the design and fabrication of a moving-coil electrodynamic geophone varies with the manufacturer, it is reasonable to represent any geophone of this type as a damped harmonic oscillator. The geophone model shown in Figure B-1 is adopted here as a fairly realistic representation. The following development is stimulated by an approach to geophone analysis described by Pieuchot (1984).

The source of the magnetic field is a ring-shaped magnet of high permanent magnetization. A coil of fine wire is suspended by a spring in the annular gap between a ring-shaped pole piece and a central cylindrical pole piece, both pole pieces being made of material of high magnetic susceptibility. The magnetic induction $B$ is radial in the annular gap but axial within the central pole piece; the overall shape of the magnetic flux lines is torroidal. Motion of the case is coupled to motion of the coil through the spring, but because the coupling is not rigid, the coil can move relative to the case. In an ideal geophone, the relative motion is restricted to be in a direction that is parallel to the axis of the geophone. The axis is taken to be vertical in the following analysis but the development for a geophone with a horizontal axis is essentially the same.

Several measures of vertical displacement are needed in order to derive the equation of motion for the coil. The reference frame shown in Figure B-1 with origin $O$ is fixed with respect to an undisturbed portion of the earth. The top of the coil (point $A$) is chosen as a reference point indicating the vertical position of the coil. When the spring is undeformed (i.e., in an unextended and uncompressed position), point $A$ is aligned with the origin $O$ on the vertical $z$-axis of the reference frame, and the reference point on the case (point $B$) is also aligned with origin $O$. Under static conditions in the earth’s gravitational field, the weight of the coil causes the spring to sag so that point $A$ is located at position $\Delta$, whereas point $B$ remains aligned with origin $O$. During ground motion, the vertical displacement of the case $u_B(t)$ is measured by the position of point $B$ along the $z$-axis, and similarly the vertical displacement of the coil $u_A(t)$ is measured by the position of point $A$ along the $z$-axis.

Of course, if the geophone is rigidly attached to the earth, then $u_B(t)$ is also the particle displacement. The inertia of the coil and the nonrigid coupling between coil and case means that, in general, $u_A(t)$ will differ from $u_B(t)$. That difference between them is the vertical displacement of point $A$ (i.e., the coil) relative to point $B$ (i.e., the case), and is represented here by the symbol $u(t)$:

$$u(t) = u_A(t) - u_B(t).$$  \hspace{1cm} (B-1)
where \( u(t) \) is called the relative coil displacement. Consideration of the definitions of points \( A \) and \( B \) indicates that \( u(t) \) is also the extension of the spring. It will prove convenient to define one additional vertical displacement function:

\[
y(t) = u(t) - \Delta,
\]

where \( y(t) \) is the spring extension in excess of its static extension under the weight of the coil.

According to Newton’s second law, the sum of the forces acting on the coil must equal the mass of the coil \( m \) times its acceleration in the chosen reference frame:

\[
\text{Sum of Forces} = m \frac{d^2y(t)}{dt^2}.
\]

(B-3)

The forces acting on the coil are summarized as follows:

- a constant gravitational force directed vertically down and of magnitude \( mg \) where \( g \) is a positive number giving the gravitational acceleration
- the spring force, which is directed opposite to \( u(t) \) with magnitude \( K |u(t)| \) where \( K \) is a positive number called the spring constant
- a mechanical damping force directed opposite to the velocity \( \frac{du(t)}{dt} \) and of magnitude \( D |\frac{du(t)}{dt}| \) where \( D \) is a positive number called the mechanical damping coefficient
- an electromagnetic damping force of magnitude \( 2\pi n I(t) \) that opposes the motion of the coil, where \( I(t) \) is a positive number giving the current induced in the coil by its motion relative to the magnetic field, \( r \) is the radius of the coil, and \( n \) is an integer giving the number of turns in the coil

The mechanical damping force represents all viscous effects as the spring/coil system moves in the air-filled annular gap. The electromagnetic damping force is the force exerted on the electrical current \( I(t) \), and is computed by integrating \( dF = (\mathbf{d} \times \mathbf{B})I(t) \), where \( \mathbf{d} \) is a differential length element along the coil in the direction of the current. As a consequence of Lenz’s law, the current direction will be such that the magnetic force instantaneously opposes the coil motion. In this case, the vector cross product \( \mathbf{d} \times \mathbf{B} \) gives a force that is vertical (up or down depending on the current direction) at every point of the coil.

All of the time-dependent forces acting on the coil are given above in terms of the relative coil displacement \( u(t) \) except the electromagnetic damping force. It is therefore necessary to express the current \( I(t) \) in terms of \( u(t) \). This is accomplished by first computing the magnitude of the induced electromotive force \( \epsilon(t) \) in the coil using Faraday’s law:

\[
\epsilon(t) = \frac{d\Phi(t)}{dt},
\]

(B-4)

where \( \Phi(t) \) is the total magnetic flux through the coil and \( \epsilon(t) \) is such that it produces a current that counteracts the flux change. Because the flux can change only by moving the coil relative to the case, the flux must be a function of \( u(t) \):

\[
\Phi(t) = \Phi(u(t)).
\]

(B-5)

Substituting the flux in equation (B-5) into equation (B-4), and applying the chain rule for derivatives, gives the electromotive force in terms of the relative coil displacement:

\[
\epsilon(t) = C_0 \frac{du(t)}{dt},
\]

(B-6)

where \( \Phi(u) \) is assumed to be linear for small enough \( u(t) \) that \( |d\Phi(u)/du| \) can be approximated by a positive transduction constant \( C_0 \) with SI units of V per m/s.

\[\text{Reference frame} \]

Figure B-1. Vertical moving-coil electrodynamic geophone attached to the earth with a spike. The vertical axis of the reference frame is fixed with respect to an undisturbed portion of the earth. Reference point \( A \) at the top of coil and reference point \( B \) on the case both align with origin \( 0 \) when the spring is in undeformed state. Position \( \Delta \) is static position of \( A \) when spring deforms under the weight of the coil. Resistances \( R_A, R_L, \) and \( R_S \) are the input resistance (amplifier impedance) of the seismograph, resistance of the line between the geophone and the seismograph, and the geophone shunt resistance, respectively.
The current generated by \( \varepsilon(t) \) depends on the resistance \( R_c \) and inductance \( L_c \) of the coil, and the external circuit attached to the geophone terminals. Analysis of the external circuit shown in Figure B-1 gives the following result:

\[
\varepsilon(t) = R_c I(t) + R_E I(t) + L_c \frac{dI(t)}{dt},
\]

(B-7)

The resistance \( R_E \) is the equivalent resistance of the external circuit:

\[
R_E = \frac{R_E (R_A + R_L)}{R_c + R_A + R_L},
\]

(B-8)

where \( R_E \) is the geophone shunt resistance, \( R_A \) is the input resistance (amplifier impedance) of the seismograph, and \( R_L \) is the resistance of the line between the geophone and the seismograph. If the inductance term in equation (B-7) is small relative to the resistance terms, then equations (B-6) and (B-7) can be combined to give the desired expression for the current \( I(t) \) in terms of the relative displacement \( u(t) \):

\[
I(t) = \left( \frac{C_0}{R_c + R_E} \right) \frac{du(t)}{dt}.
\]

(B-9)

Neglecting the inductance of the coil, and representing the external circuit as a pure resistance \( R_E \), are simplifications that do not undermine the fundamental results of the analysis.

The equation of motion of the coil is obtained according to equation (B-3) by summing the four forces listed above and equating the sum to the product of the coil mass and acceleration

\[
m g - K u(t) = \left( D + \frac{C_0 2 \pi \nu n |\mathbf{B}|}{R_c + R_E} \right) \frac{du(t)}{dt} = m \frac{d^2 u_A(t)}{dt^2},
\]

(B-10)

where equation (B-9) is used for \( I(t) \) in the electromagnetic damping force. Because \( m g = K \Delta \) and the time derivatives of \( u(t) \) and \( y(t) \) are identical, the equation of motion (B-10) can be written in terms of the spring extension \( y(t) \):

\[
mg \Delta - K \Delta \frac{dy(t)}{dt} = m \frac{d^2 y(t)}{dt^2} + \left( D + \frac{C_0 2 \pi \nu n |\mathbf{B}|}{R_c + R_E} \right) \frac{dy(t)}{dt} + Ky(t) = -m \frac{d^2 y_B(t)}{dt^2},
\]

(B-11)

where definition (B-1) has been used to substitute for \( u_A(t) \). The dependent variable in equation of motion (B-11) can be converted from \( y(t) \) to the voltage \( V(t) \) across the seismograph input by multiplying the input resistance \( R_A \) by the current \( I(t) \) to obtain the following relation:

\[
V(t) = \left( \frac{-R_A C_0}{R_c + R_E} \right) \frac{dy(t)}{dt},
\]

(B-12)

where the current \( I(t) \) is from equation (B-9). The voltage sign convention is chosen in equation (B-12) so that a positive voltage corresponds to an upward motion of the coil (downward motion of the case). Differentiating equation of motion (B-11) once with respect to \( t \) and applying equation (B-12) gives the geophone equation:

\[
\frac{d^2 V(t)}{dt^2} + 2 \omega_0 \frac{dV(t)}{dt} + \omega_0^2 V(t) = C \frac{d^2 v_B(t)}{dt^2} = C \frac{d^2 y_B(t)}{dt^2},
\]

(B-13)

where \( v_B(t) \) and \( a_B(t) \) are the velocity and acceleration of the case, respectively:

\[
v_B(t) = \frac{du_B(t)}{dt}
\]

and

\[
a_B(t) = \frac{dv_B(t)}{dt}.
\]

The constants \( \omega_0, h, \) and \( C \) are called the natural frequency, total damping constant, and total transduction constant, respectively, and are defined as follows:

\[
\omega_0 \equiv \frac{K}{\sqrt{m}}, \quad \text{and} \quad h \equiv \frac{1}{2m \omega_0} \left( D + \frac{C_0 2 \pi \nu n |\mathbf{B}|}{R_c + R_E} \right),
\]

(C-15)

As mentioned in the main text, the geophone equation (B-13) is important because it relates the voltage observed at the seismograph input to the motion of the case. The motion of the case is a proxy for particle motion if the geophone is firmly attached to the earth.
Near-Surface Seismology: Surface-Based Methods


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Supplementary References
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Other applications or methods


