

Improved Water Table Dynamics in Block-Centered Finite-Difference Flow Models.

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ABSTRACT

The standard formulation of a block-centered finite-difference model, such as MODFLOW, uses the center of the cell as the location of a cell node. Treating the node location of partially saturated cells to be at the water table improves the accuracy of a MODFLOW simulation. Changes to only a few lines are needed. Improvement of MODFLOW-2000 simulations in which the location of nodes dynamically follow the water table are demonstrated through comparisons to the analytically based calculations of the WTAQ code. The changes introduce a non-linearity into the solution of the flow equations that results in slightly slower convergence of the flow solution; convergence is 7% slower in the presented demonstration.

INTRODUCTION

MODFLOW (McDonald and Harbaugh, 1988) is based on a three-dimensional block-centered finite-difference formulation. The flow properties of each cell are specified by a set of hydrologic parameters that are constant within each cell, though this is relaxed somewhat with the hydrogeologic-unit flow package. The finite-difference equations of flow through the subsurface use a single value to describe the hydraulic head within a cell. The equations are formed using the approximation that the cell head value is the head at a specific location, the cell's node. The term "block-centered" indicates that the node is located at the center of the cell.

Simulations of fluid flow near a water table are improved if the node of a cell that contains the water table is located at the water table rather than at the center of the cell. The location of the node should dynamically follow water table movement. MODFLOW-2000 (Harbaugh et al., 2000) uses the water table elevations to calculate the horizontal hydraulic conductance but not to position the node location. Since the water table elevations are already available in the relevant subroutines, a minor change to the code to reposition the cell node at the water table results in improved accuracy.

THE CASE FOR DYNAMIC WATER TABLE NODES

In the case of completely saturated layers, a change in head causes a change in porosity due to the compressibility of the aquifer which acts to dampen the head change. The porosity change occurs throughout a model cell and is approximated well by treating the change as if it all occurred at the center of the cell. MODFLOW approximates a partially saturated cell as fully saturated below the water table and unsaturated above it. The change in the saturated porosity due to a change in head is dominated by the movement of the water table rather than the change due to compressibility.

I find a conceptual sleight-of-mind helps illustrate the difference. The concept of a physical change in the void space is replaced by the concept of fluid addition or removal. The concept is consistent with the phrase "water coming out of storage" that is often used to describe the effects of specific storage. The equations of flow would be identical if instead of a decrease in void space, an addition of fluid occurred, with a volume equivalent to the decrease in void space. With respect to specific storage, this source of addition or removal of water would be distributed throughout the cell volume. To approximate the source as occurring at the center of the cell is appropriate.

In the case of specific yield, the change in void space available to the fluid occurs above the water table, which is approximated as occurring at the water table in a fully saturated flow code. For a change in the water table position, the fluid source should be positioned at the water table since it is there that the change in void space occurs. Approximating the source at the center of the cell is equivalent to instantaneous transport of the "added or removed water" from the water table to the center of the cell.

Briefly, the cell-to-cell conductance is inversely related to the distance between nodes. For example, if both cells have the same hydraulic conductivity and each are of equal thickness, then positioning the node at a water table near the top of the upper cell results in two-thirds the cell-to-cell conductance. If the water table is near the bottom of the upper cell then the conductance between the cells would be twice the conductance of nodes positioned in the center of the cells.

IMPLEMENTATION

Only a small change in coding is needed to implement a water-table-following node in the layer-property flow package. In the subroutine SGF1LPF1VCOND (gwf1lpf1.f, dated 13JAN2000) the vertical conductance between two cells is calculated. The variable BOVK1 represents the distance from the node to the cell boundary divided by the vertical hydraulic conductivity for the upper cell. In this calculation the distance from the center of the cell to the boundary (half the cell thickness) is replaced by the distance from the bottom of the cell to the water table. The calculation of BOVK1 changes from

$$\text{BOVK1} = 0.5 * (\text{BOTM}(\text{J}, \text{I}, \text{LBOTM}(\text{K}) - 1) - \text{BOTM}(\text{J}, \text{I}, \text{LBOTM}(\text{K}))) / \text{HYC1}$$

to

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IF( HNEW(I,J,K) .LT. BOTM(J,I,LBOTM(K)-1) ) THEN
  BOVK1 = (HNEW(I,J,K)-BOTM(J,I,LBOTM(K)))/HYC1
ELSE
  BOVK1=0.5*(BOTM(J,I,LBOTM(K)-1)-BOTM(J,I,LBOTM(K)))/HYC1
ENDIF
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The variable BOTM(J,I,LBOTM(K)) is the bottom of the cell and HYC1 is the vertical hydraulic conductivity of the cell.

Implementing a water-table-following node in the hydrogeologic-unit flow package, while straightforward, is not as simple as with the layer property flow package. The effective vertical hydraulic conductivity of a cell depends on the water table elevation because the relative contribution of the hydrogeologic-units may change as the water table elevation changes. Thus both the distance between nodes and the hydraulic conductivity must be adjusted.

The parameter estimation process of MODFLOW-2000 (Hill et al., 2000) should be updated to reflect the changes to the conductance. I have not investigated the required modifications to the sensitivity calculations for water-table-following nodes.

COMPARISON TO WTAQ

The influence of the position of the water table node using MODFLOW may be compared to WTAQ3 (Barlow and Moench, 1999) calculations for a hypothetical homogeneous aquifer. WTAQ3 provides a numerical implementation of analytic solutions (Moench, 1997) to homogeneous water-table aquifer responses to pumping from a partially penetrating finite-diameter well. The properties of the hypothetical aquifer used for the comparison are isotropic hydraulic conductivity of 0.0001 m/s, specific storage of 0.0001 per m, specific yield of 0.38, and an aquifer thickness of 18 m.

The pumping well has a radius of 0.0508 m and is screened over an interval of 8 m to 12 m below the water table. Two MODFLOW model definitions are used in the comparison. Both have the same 43 rows and 45 columns. The horizontal grid spacing was constructed such that nodes occurred close to the following distances from the pumping well: 3.52 m, 7.08 m, and 22.1 m. The boundaries of the model are far enough away that there is no appreciable response during the 10,000 s pumping period. One model has 21 layers the other 26. Each layer in the 21-layer model is 1 m thick except for 0.5 m thick layers above and below pumping interval, and at the top and bottom of the model. The 26-layer model has a fine grid near the water table. In the 26 layer model, the uppermost layer is 0.1 m thick and there are eight layers in the upper 3 meters.

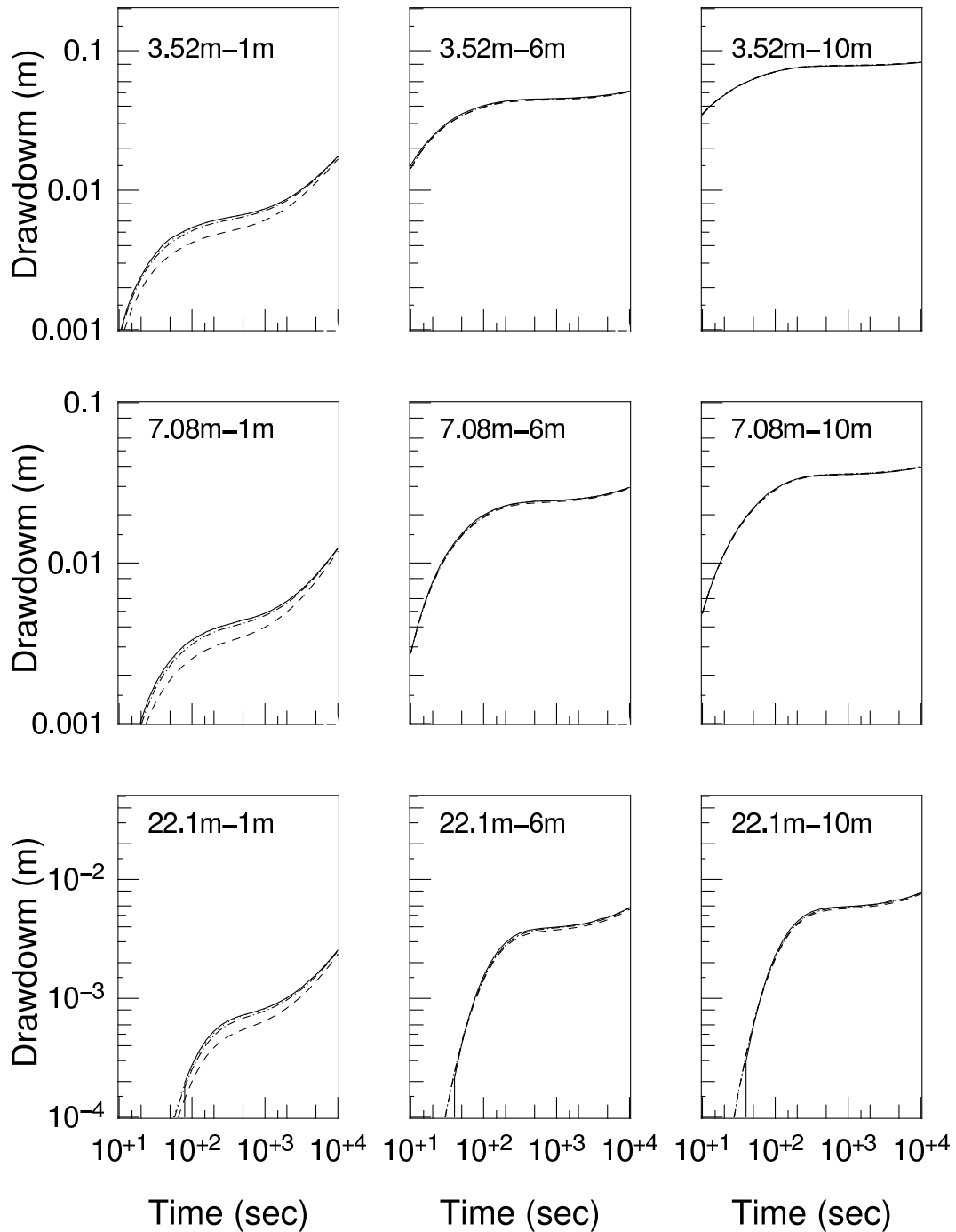


Figure 1 Semi-log comparison of drawdown at various locations. WTAQ calculation (solid), unmodified MODFLOW with 21 layers (dashed), unmodified MODFLOW with 26 layers (dot-dash), modified MODFLOW with 21 layers (dotted). The dotted line overlays the solid line at this scale.

Figure 1 presents calculations of drawdown at distances from the pumping well of 3.52 m, 7.08 m, and 22.1 m at depths of 1 m, 6 m, and 10 m below the water table. Four sets of data are presented in each panel: WTAQ calculations are shown as a solid line, dashed curves plot the 21 layer model without the modification, dash-dot curves are for the 26 layer model, and the dotted curves show the results from the 21 layer model with the water table modification. At all distances from the well, the models are in close

agreement with WTAQ at depths well below the water table. A disagreement is apparent in the figures at nodes 1 m from the initial water table. The differences are largest at times when the drawdown is first impacted by the water table decline - causing a brief plateau or delayed yield period in the drawdown. The calculations merge again after the plateau region. The water table decline is fast with respect to the time scale after the plateau region and water table dynamics become unimportant.

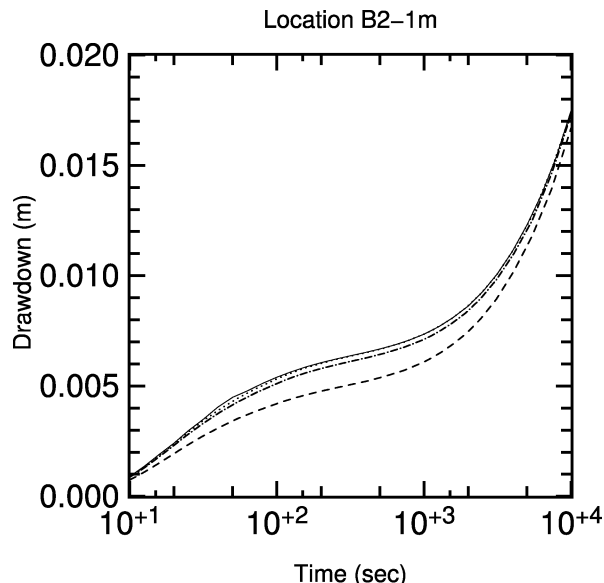


Figure 2 Semi-log comparison of drawdown at a depth of 1 m below the water table 3.52 m away from the well. WTAQ (solid), unmodified with 21 layers (dashed), unmodified with 26 layers (dot-dash), and modified with 21 layers (dotted).

was modified to calculate the conductance for a water table bearing model cell based on the water table position instead of the center of the cell. Comparisons of drawdown simulations from both the modified and unmodified codes with the analytically based WTAQ code reveal that: 1) fine vertical discretization of layering near the water table improves the accuracy of the MODFLOW calculations but at a cost of longer execution times; 2) dynamically following the water table can be more accurate than a fine vertical discretization and at less cost in terms of execution times. Acknowledgments: This work was supported by U.S. Army Research Office grants DAAH04-96-1-0318 and DAAD19-00-1-0454 and EPA grant X-970085-01-0. CGISS publication number 0122.

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Figure 2 shows the region of greatest disagreement: the plateau region of the location nearest to the water table and closest to the well. In this figure, the differences between model calculations are more clearly presented. The drawdown of the 21 layer model is approximately 20% less during this period. The 26 layer model reduces the maximum error to about 4% and the modified 21 layer model has a barely discernible under-prediction of drawdown. The calculations merge again after the plateau region. The water table decline is fast with respect to the time scale after the plateau region and water table dynamics become unimportant.

For the 21-layer geometry, the water table modification slows the execution speed of the model by 7%. Using the unmodified code with 26 layers instead of 21 layers requires 31% more time to reach a solution.

CONCLUSION

A more accurate simulation of water table dynamics with block-centered finite difference simulation of fluid flow can be accomplished if the vertical position of the nodes in the upper cells dynamically follow the water table. The USGS MODFLOW-2000 code